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BRICKS

Building Resources for Integrated Cultural Knowledge Service

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M2.5.2 - Minutes of the workshop on "The Problem of Diagrams and Drawings Criticism in Mathematical Texts"

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Contents

Chapter 1

Introduction

1.1 General description

The workshop took place in Pisa, Department of Mathematics "Leonida Tonelli" 25–27 November 2004.

Organizing Committee

Veronica Gavagna, Paolo Mascellani, Pier Daniele Napolitani

Scientific Committee

Ottavio Besomi, Enrico Giusti, Pier Daniele Napolitani, Carlo Maccagni

Thanks for help to Department of Mathematics of Pisa and Domus Galilaeana.

The detailed programme of the workshop is in § A.

1.2 How the workshop's idea was conceived

The development of the *Maurolico Project* is organized into different steps. The first one consists in having at least the complete transcription of all texts, possibly originating from a collation of all available witnesses. To do this, every Maurolico's editor has asked for help to the methods and techniques of textual criticism.

On the other side, having to deal with the critical edition of the mathematical diagrams, people have noticed the lacking of shared and general criteria. Critical editions of mathematical diagrams are still in lack of standardized criteria able to satisfy both scientific and philological needs.

Every editor working to the critical edition of a mathematical text has necessary realized there is no a system of general and shared criteria which underlie the publication of diagrams.

In fact, getting a look into the circulating critical editions containing mathematical diagrams many different editorial strategies are available: some editors have chosen to reproduce the original drawings without any editorial interference (even if the same editor has made intervention in the text) some others have chosen to re-build *ex novo* the diagrams, giving in this way a strong editorial interpretation. Between these two limits there is a range of different solutions in restoring the original figures. But there are no traces of a system of general criteria. In other words, it seems that a sort of "diagram criticism" doesn't exist, yet.

On the other side, it's clear that a critical method of editing mathematical diagrams is a real need of every critical editor of mathematical writings. So, the lacking of the so-called "diagram criticism" and the shared need to define it, incited the organizers to arrange this workshop with the support of the European BRICKS Project.

1.3 Structure of the workshop

The organizers have rejected the hypothesis of organizing a traditional workshop where the speakers give a talk followed by 10-15 minutes of discussion. This workshop was in fact focused mainly on the discussions arisen by the cases-study.

That's why the speakers have been invited to describe briefly their case study, circumscribed to a diagram or a small group of diagrams, emphasizing the relevance of the diagram in understanding the mathematical text and the relevance of the chosen diagram from a critical point of view. All information useful to set the case study against its historical background have been written in the abstracts given by the authors some days before the beginning of the workshop. In this way, the participants could read the abstracts, in order to let the speakers focus their attention on the main features of the diagram involved (from a philological and/or mathematical point of view) and explain the adopted solution. From this point of view, each talk has been represented the starting point of a discussion among scholars with different skills. The organizers have invited historian of science and art, historian of mathematics, philologist and computer experts.

The programme has been planned following a chronological principle (for the detailed programme, see § A). The opening case studies were devoted to Italian Renaissance, regarding the Leon Battista Alberti's edition of the *Ludi mathematici* and the Piero della Francesca's *Libro d'abaco*. The following morning has been devoted to the Euclidean and Spherical tradition in the Middle Ages and in the Renaissance and to the problem of editing mathematical diagrams in Arabic and Chinese texts. The afternoon session has been devoted to authors of XVII-XVIII century: Leibniz, Bernoulli and Desargues. The last session was opened by the case study regarding Galileo and went on with a general discussion finally summed up by Pier Daniele Napolitani.

1.4 Goals of the workshop

The first realistic goal of the workshop was, obviously, only to make a reconnaissance, a first exploration of the wide and complex problem of editing mathematical diagrams. In particular, the organizers wanted to

- focus the most relevant features which underlie the definition of a critical apparatus of diagrams
- verify if it is possible define only one shared system of criteria or, if it's better specify different system of criteria related to particular kind of critical editions.

Chapter 2

Topic at issue

2.1 General topics

2.1.1 Connections between mathematical text and diagram

Is well known that history of mathematics means, essentially, history of mathematical texts. But a mathematical text can be regarded as a pure text or not? Everybody can answer in different manners, but has to admit at least a peculiar feature, namely the relationship between mathematical texts and diagrams.

Mathematical diagram plays an important role for understanding the related text, it represents an essential part of a proof, especially of a geometrical proof. But is it true establish that mathematical text and mathematical diagram lie on the same level of comprehension?

Given a mathematical text is always possible building the related diagram[1], but given a geometrical diagram, the reconstruction of the related mathematical text is – generally – impossible. This consideration leads to suppose a sort of supremacy of the text on the diagram.

2.1.2 Mathematical diagrams and descriptive codes

Many speakers have emphasized how the criteria which underlie the construction of mathematical diagrams are not absolute, depending on descriptive codes changing along the time.

Modern mathematicians, for example, build "general diagrams". So, if it is written "The heights of a given triangle meet themselves in one point" the modern mathematician draws a scalene triangle. A mathematician lived in ancient Greece, even if the text was referred to a general case, could draw an isosceles or equilateral triangle. Ken Saito explained that the Greek diagrams are "standardized". In other words, an angle becomes a right angle if it can be, two lines are drawn equal if they can be equal, a segment of a circle becomes a semicircle if it can be. Thus, there appear more squares, rectangles, isosceles and equilateral triangles in the diagrams of Greek manuscripts than in their texts. Heiberg "generalized" these diagrams, for example, making two lines different if they are not always equal, followed by all the current translations. However, the "generalization" seems to be a recent tendency.

Furthermore, metrical correctness is not so important in Greek diagrams and even the distinction between straight lines and curved lines is not of absolute importance.

[1]Of course, the same mathematical text could originate different diagrams in different historical periods, because the construction of drawings depends on descriptive codes in use.

Other features influence a descriptive code. For example, the distribution of the letters in diagrams contained in Arabic manuscripts is specular to the Western codes and reflects the different way of writing.

2.1.3 Connections between diagrams and the *stemma codicum*

The analysis of diagrams belonging to different witnesses is a very useful tool to build the *stemma codicum*.

The case study presented by K. Saito, for example, is devoted to proposition III.25 of Euclid's *Elements* (to complete a given segment of a circle). The three diagrams in Heiberg's edition of the Euclidean *Elements*, correspond to the three cases in the proof: the given segment being less, equal, greater than a semicircle. However, the manuscripts present only one diagram in the column where the text and the diagram appear, and the three diagrams according to three cases appear in margin. Furthermore, the diagram in the column seems to serve for all the three cases. So, it is possible that the division of the proof into three cases is a result of later intervention, and the original proposition presented the proof only once in the text, accompanied by one figure applicable to three cases.

Pascal Crozet has shown how an hypothesis of *stemma codicum* based on the comparison and the analysis of textual variants has become stronger comparing "graphical variants".

2.1.4 Autographs in textual tradition

There are different rules which underlie the editing of manuscripts or printed texts. It seems that this distinction still survives also in the editing of diagrams.

When the critical editor publishes an autograph text, he tries to emphasize the author's style in writing, keeping on unusual or fluctuating readings.

How the critical editor can define the rules to edit sketches and drawings (may be corrected several times) contained in an autograph text? One of the most complex case is the edition of Leibniz's manuscripts. Most of the diagrams were hastily sketched in pen and ink, only few are partly or completely constructed, many contain corrections or alterations. Sometimes there are elements (lines or points) in a drawing that are not referred to within the accompanying text. Or there are texts that refer to drawings in other manuscripts that do not exist any more or simply cannot (yet) be identified.

Which criteria must be adopted by the critical editor? How is it possible to choose which drawings are suitable for publication and which ones not?

Siegmund Probst has explained that the conventions used in the academy edition suggest that whether a deleted figure will be edited depends largely on an evaluation of the mathematical content. If the mathematical content isn't relevant, the deleted figure will only be mentioned in a footnote. But, of course, this convention is far from giving a real solution to each problem arisen in the editing of Leibniz's manuscripts.

2.1.5 Is it possible define only one system of shared criteria?

It seems it is not possible give an affirmative answer to the possibility of defining a unique system of criteria that could be called "diagram criticism" and underlies the editing of mathematical diagrams.

It seems more realistic to define different "diagram criticism" related to particular historical periods, e.g. Greek mathematics, or specific cultures, e.g. Arabic or Chinese, or connected to well-defined philological features, e.g. autograph witnesses.

2.2 Collating diagrams

The main problem of editing mathematical diagrams is to deeply understand what does it mean *collating mathematical diagrams*. In other words it is necessary to establish when two or more diagrams can be regarded as equal or, when one diagram can be considered a graphical variant.

2.2.1 How to define graphical variants

A convincingly definition of graphical variant can be originated only by the analysis of different situations.

For example, if the witness A (autograph) contains a circle drawn by a compass and the witness B (autograph) contains in the same point a sort of oval drawn by hand, it is probable that the two diagrams must be regarded as equal. But if A and B are printed text, and the former presents a circle and the latter an ellipse, it's quite improbable that the two diagrams must be regarded as equal.

Furthermore, two diagrams that are perfectly equal unless added or erased lines, must necessarily be considered graphical variants.

The suggestion arisen during the general discussion is to define two or more diagrams as 'equal' if they are topologically equivalent.

2.2.2 How to describe graphical variants

It is necessary to develop a suitable language to describe the critical apparatus related to diagrams.

As well as in the critical text, the editor inserts a particular kind of parenthesis to point out an addition or deletion, it will be useful to define a set of shared symbols to indicate addition or deletion of lines or of geometrical elements. But, in this case, the symbol must indicate also the position of the line/element within the diagram.

2.3 Advantages of electronic editions

The electronic edition let the editor to accumulate many information and to choose which of them will be available to the reader.

To give an example, the Mauro-TeX language, developed for the *Maurolico Project*, permits not to appear in the critical apparatus the so-called "trivial variants". These variants are not lost: they still survive in the file source of the edition and they can appear in the critical apparatus whereas the editor decides it. In this way, the critical apparatus will be "definitive" only when the electronic edition will become a printed edition.

So, these are the main advantages in choosing an electronic edition:

- it's possible to store many information and to select the information available to the reader;
- it's possible to postpone the editorial chooses, getting them definitive only in the moment of printing.

2.4 Future prospects

It seems not realistic, as it's written in § 2.1.5, the hypothesis of defining a unique system of shared criteria, named "diagrams criticism".

That's why the organizers suppose it will be useful arrange future seminars or workshop circumscribed to particular historical periods or to specific philological principles.

Further seminars would be devoted to the crucial problem of defining and describing graphical variants (see §§ 2.2.1 e 2.2.2).

Appendix A

Programme

November 25th, 2004

Department of Mathematics

15.00 – 17.30

- Welcome address
- Presentation of BRICKS' Project (Paolo Mascellani)
- Presentation of the Workshop (Veronica Gavagna)
- Inaugural conference
  CARLO MACCAGNI (Genova)
  Diagrams Criticism in Mathematical Textual Traditions

17.45 – 20.00

- VLADIMIRO VALERIO (Venezia)
  "Egl'è uno triangulo ABC che AB è 15, BC 14, AC 13. . . " Remarks on the proportion and measure of geometrical figures from Piero della Francesca's Abaco
- FRANCESCO FURLAN (Paris)
  Sur l'ecdotique des figures scientifiques: le cas des "Ex ludis rerum mathematicarum" d'Alberti

November 26, 2004

Department of Mathematics

9.00 – 11.30

- KEN SAITO (Osaka)
  The diagrams in Codex P of Euclid's "Elements"

- ALESSANDRA SORCI (Roma)

  The case XIII.16 of Euclid's "Elements". Some remarks on the iconographical traditions and their editions

- MICHELA MALPANGOTTO (Bari)

  Some remarks about the diagrams of "Theodosii Sphaericorum libri tres"

- PASCAL CROZET (Paris)

  Editer les figures des manuscrits arabes des géométrie: l'exemple d'al Sijzî

- KARINE CHEMLA (Paris)

  Editing the earliest extant mathematical figures from China

15.00 – 17.30

- FRANCO GHIONE (Roma)

  Diagrams in Menelaus' Theorem

- PATRICIA RADELET-DE-GRAVE (Louvain)

  L'édition des figures manuscrites des Bernoulli

18.00 – 20.00

- SIEGMUND PROBST (Hannover)

  Editing Mathematical Drawings from Leibniz's Manuscripts

- HARTMUT HECHT (Berlin)

  The Dynamics of Leibnitian Drawings and some Reflections on its Representation

November 27, 2004

Department of Mathematics

9.00 – 11.00

- OTTAVIO BESOMI (Zürich)

  Riflessioni intorno all'edizione delle figure che accompagnano il "Saggiatore" di Galileo Galilei

- Discussion

11.15 – 13.00

- General discussion on the possibility to define a critical method in the editing of mathematical diagrams

Appendix B

Abstracts

B.1 APPENDIX 2: Abstracts

B.1.1 V. Valerio, *Remarks on the proportion and measure of geometrical figures from Piero della Francesca's Abaco*

During the recognition of the geometrical figures of the Abaco by Piero della Francesca, arose the question of how to dimension the "critical" drawing. The Commission for the publication of the scientific writings by Piero della Francesca, following my suggestion, decided to realize both "diplomatic" and "critical" drawings in the same dimension as the original. Thus it was possibile to show on one side the exact dimension and shape of the figures ("diplomatic" drawings), and on the other to compare the "diplomatic" drawing with the "critical" one in order to evidentiate the difference among them. While the "diplomatic" drawing is a slavish copy of the original, putting in evidence all the lines used for its accomplishment-i.e. blank lines traced with a smooth iron stik, arc of circles draught to determine with their intersection points and vertices of polygons, etc.- the "critical" drawing must follow exactly what is established in the text, such as letters, measures of the sides of polygons, height, radius of circles circumscribed and other graphical operation used to solve the problem. The sample shown here stresses the difficulty of making the "critical" drawing when the figure does not fit the measures reported in the problem; the exercise taken into account is numbered 402, according to the critical text, and is described at f. 80r as follow:

> Ma quando non fusseno equali commo sesse uno triangulo che fusse AB 15, BC 14, AC 13 et BC fusse la basa ch'è 14, montiplica in sé fa 196; montiplica AB ch'è 15 in sé fa 225, giogni con 196 fa 421; montiplica AC ch'è 13 in sé fa 169, trallo de 421 resta 252. Parti per lo doppio de la basa BC ch'è 14 sirà 28, ne vene 9 et 9 è da B al puncto dove cade il catecto. Montiplica 9 in sé fa 81 et montiplica AB ch'è 15 in sé fa 225, tranne 81 resta 144, la sua Rx è il catecto ch'è 12. E questo modo serve ad omni triangulo.

What I find relevant for my paper is that, notwithstanding Piero has drawn a scalene triangle, the lenght of the sides are not in the right proportion 13, 14, 15, as suggested in the text.

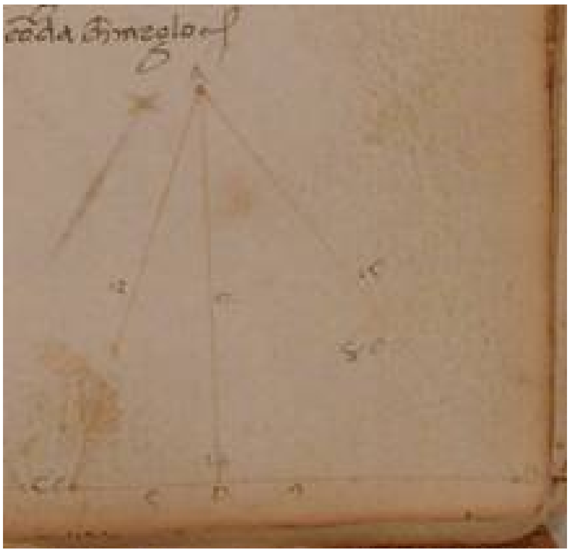


Fig. 1. Scalene triangle related to exercise n. 402, f. 80r of the
Abaco by Piero della Francesca (Bibl. Laur. Ashb. 359)

While there is no problem in tracing the “diplomatic” drawing, how to draft the “critical” one was a compelling matter and not easy to solve. In fact, being the sides not in the right proportion-that means that triangle is not drawn on scale-the question is which side of the same triangle to chose as starting point? Or, in other words, which measure is to be considered as the right one, as we wanted to make “diplomatic” and “critical” drawings in the same dimension? In this case, as in other folios of the Abaco, I paid attention to the construction of the drawing trying to deduce the temporal sequence of the lines. The first lines drawn is assumed as Piero’s starting point and should have also been mine.

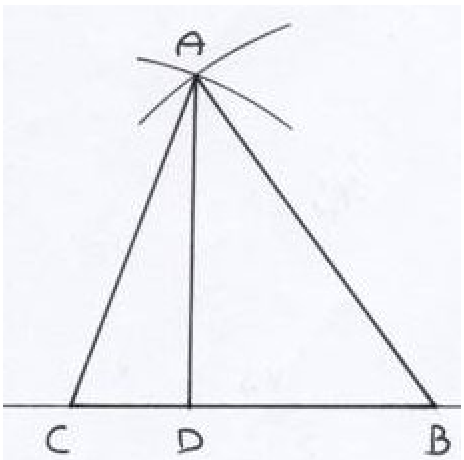


Fig. 2. “Diplomatic” drawing taken from f. 80r of the Abaco by
Piero della Francesca.

The triangle has its base drawn in ink on a line “a secco” which serves as common base to both triangles in folio 80r. Furthermore the points marked C and B they are centers of two arcs of circle whose intersection mark the vertex A. The radius of the two circle should be in the proportion 13 and 15 compared ot the base whose measure is 14. We don’t know why these sides are not in the right proportion, especially as in other exercises Piero does it, but the way of constructing the figure suggests us which line is to be considered as “base line”.

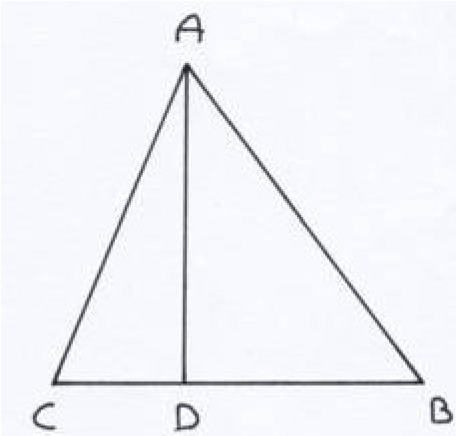


Fig. 3. “Critical” drawing taken from f. 80r of the Abaco by Piero della Francesca.

The critical drawing is then constructed using the base BC as the starting side, putting the others in the right proportion, so the diplomatic and the critical drawings are now in what we mean “the same dimension” and it is possible to compare them looking for significant discrepancies and differences.

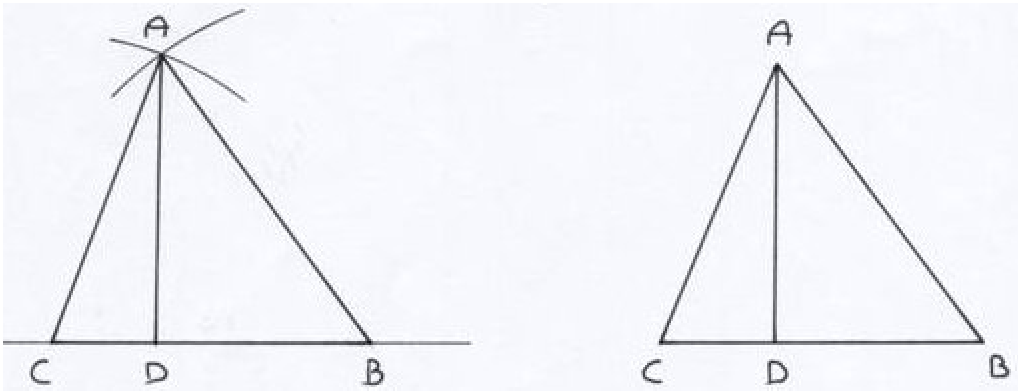


Fig. 4. Diplomatic and critical drawings of the triangle in f. 80r of the Abaco by Piero della Francesca drawn in the same dimension.

B.1.2 F. Furlan, *Sur l’ecdotique des figures scientifiques: le cas des “Ex ludis rerum mathematicarum” d’Alberti*

1. La tradition manuscrite des *Ex ludis rerum mathematicarum* d’Alberti est marquée par un nombre élevé de fautes en tous genres reflétant principalement l’inattention certaine des copistes (il s’agit d’une tradition non littéraire et, partant, relativement non savante) ainsi que le caractère “glissant” d’un texte qui accumule expressions et formules dont la proximité, si ce n’est aussi l’identité facilite grandement les sauts du même au même.

2. Les éditions existantes (y compris la plus autorisée d’entre elles, publiée en 1973 par Grayson) n’ont pas pu résoudre les nombreux problèmes posés par un texte souvent erroné et mathématiquement incohérent, les éditeurs eux-mêmes n’ayant souvent, par ailleurs, aucune conscience de la spécificité du problème ecdotique posé par la tradition d’ouvrages anciens comprenant à la fois verba et picturæ, ni des retombées épistémologiques de l’impossibilité matérielle avérée, avant l'âge de l'imprimerie et pour quiconque désirait soustraire ses propres ouvrages à un processus de défiguration certaine, de confier à des copistes-dessinateurs des figures ou dessins d'une complexité même relative.

3. Cette impossibilité radicale était bien connue d'Alberti, pour qui figures et dessins (picturæ) sont un medium d'expression généralement bien peu approprié et souvent non opportun, parfois même opposé au decorum de tout écrit scientifique; c'est pourquoi les *Ex ludis* représentent, dans le cadre de l'oeuvre écrite d'Alberti, un cas véritablement unique - qu'il est toutefois relativement aisé d'expliquer -: en effet, la présence de figures accompagnant le texte dans l'original (non conservé) de cet ouvrage est pour le moins très probable.

4. Si elle est instructive à de nombreux égards, la tradition du Problème XVIIe (sur "La mesure des grandes distances") peut aussi paraître emblématique du rapport qu'entretiennent le texte et les figures dans l'âge qui précède l'avènement de l'imprimerie. Les paragraphes suivants énoncent les principales conclusions auxquelles nous sommes parvenu en l'étudiant du point de vue qui est aujourd'hui le nôtre:

4-a le texte du Problème XVIIe a été dans un premier temps défiguré et rendu incohérent par une faute de l'archétype, au demeurant en elle-même banale - une lacune par saut du même au même dans le passage 168-25/29 du texte Grayson, passage contenant à l'origine les instructions pour la construction du second triangle (i.e. DEF);

4-b et il a été dans un deuxième temps ultérieurement défiguré ou corrompu par une tentative de correction du subarchétype (ou d'un lecteur/correcteur de l'archétype) - lequel a interpolé en ce même endroit (i.e. 168-25/29) des mots tirés de 168-13/15, passage contenant les instructions pour la construction d'un premier triangle (i.e. ABC). Cette tentative (avortée) de correction a été clairement opérée à partir exclusivement du texte (verbal) du Problème XVIIe;

4-c la presque totalité des mss. (ainsi que toutes les édd.) ont des figures à la fois incomplètes, erronées et dépourvues de sens par rapport au problème posé; il s'agit de figures qui s'efforcent pour ainsi dire de contourner les instructions, incohérentes et donc inintelligibles, données par le texte en ne présentant que les trois triangles auxquels fait explicitement référence la conclusion du texte lui-même (de manière purement intuitive, le second de ces triangles, i.e. DEF, est toutefois présenté comme approximativement égal au premier, i.e. ABC); un ex. de ces figures est donné par celle de l'éd. Grayson [fig. 3].

On ne saurait en aucun cas exclure la possibilité que l'une ou l'autre des figures de ce groupe ait été simplement copiée; cependant, il est certain que quelques-unes au moins de ces figures (et très certainement la plus "ancienne" du groupe, celle dont toutes les autres - ou une partie des autres - pourraient avoir été tirées) ont été effectivement dessinées à partir du texte incohérent de la tradition connue; la coïncidence approximative des résultats auxquels sont parvenus les différents dessinateurs doit par conséquent être considérée, du moins en partie, comme fortuite;

4-d le ms. Riccardianus 2110 a une figure [fig. 2] dont un ou deux détails (la superficie du triangle EDF, dont le côté DF semble parallèle à la droite A-Bologna) paraissent pouvoir être expliqués seulement en admettant que la figure elle-même, d'abord copiée d'après un dessin correct et complet (il pourrait même s'agir du dessin original), ait été par la suite (à la hauteur de l'archétype, ou bien du subarchétype) simplifiée par l'effacement du triangle EF-Rosa (et, partant, par l'imposition d'une coïncidence des triangles EDF et ED-Rosa que semblent d'une certaine façon exiger les instructions du texte corrompu de l'archétype);

cette simplification est sans doute en elle-même une preuve de l'influence exercée par le texte (dans un point donné de la tradition, si ce n'est dès l'origine) sur la figure elle-même;

4-e le ms. Typ. 422/2 de la Harvard College Library présente une figure [fig. 1] très originale mais, en même temps, géométriquement absurde par rapport au problème qu'elle est appelée à illustrer; elle a été clairement (et intégralement) dessinée à partir du texte corrompu de la tradition connue: cette figure représente donc indubitablement un petit exploit du dessinateur, qui a tout mis en oeuvre pour essayer de respecter à la lettre les instructions de ce texte incohérent; elle est en elle-même une preuve formelle du fait que, pour les dessinateurs au moins, le texte vient inéluctablement avant les figures appelées à l'accompagner ou à l'illustrer - et il revêt de ce fait une toute autre importance.

5. Par ailleurs, dans la tradition du Problème XVIIe des Ex ludis il n'y a aucune trace de tentatives (réussies ou non) de correction du texte à partir des figures; cela ne peut sans doute s'expliquer qu'en admettant le bien-fondé d'une ou de plusieurs des affirmations suivantes - qui, toutes, ressortissent à une même hypothèse: a) si le texte est toujours copié, les figures sont dès le départ (ou bien surtout au départ) à chaque fois dessinées à partir du texte de la copie ainsi obtenue; b) le rapport du texte aux figures et des figures au texte est toujours à sens unique; pour des raisons à la fois techniques et culturelles, donc épistémologiques, il ne saurait en effet s'inverser: les figures dépendent toujours (ou sont toujours susceptibles de dépendre) du texte, la réciproque n'étant jamais admise; en somme, c'est le texte qui engendre les figures, et ce n'est jamais l'inverse; c) copiste et dessinateur font deux: le dessinateur n'intervient qu'une fois le travail du copiste achevé, parfois peut-être même en un lieu différent de celui où le texte a été transcrit; en un sens, le dessinateur intervient donc a posteriori: il ne dispose pas, ou pas toujours, des figures du modèle.

6. Si des figures sont toujours présentes dans les mss. des *Ex ludis* qui nous restent, leur nombre peut en revanche varier; tout en étant portées par différents témoins, certaines d'entre elles sont donc absentes de l'un ou de l'autre ms.; par conséquent et bien qu'il ne soit pas possible d'émettre l'hypothèse d'un original sans figures, il semble qu'il faut admettre que la tradition manuscrite privilégie nettement le texte (verbal), et que les figures ne sont conçues que comme des illustrations (au sens propre du terme), parfois même comme un complément avant tout esthétique.

7. Il résulte de tout cela que les figures qui nous ont été transmises par les différents mss. des *Ex ludis* n'ont qu'une très faible chance de remonter à Alberti ou, à tout le moins, de refléter fidèlement celles qu'il a dû concevoir et dessiner; par conséquent, et sans rien enlever à l'intérêt historique et scientifique - parfois évident ou indiscutable - des picturæ conservées, le devoir de l'éditeur consiste à dessiner ou faire dessiner des figures qui soient une illustration effective du texte qu'il a lui-même établi, plutôt qu'à reproduire les figures ou les dessins de tel ou tel ms.

La dernière figure que nous proposons [fig. 4] a été dessinée à partir de ces principes et sur le fondement d'une étude approfondie de la tradition des Ex ludis qui a permis à Pierre Souffrin et à nous-même de proposer un texte du Problème XVIIe enfin géométriquement cohérent et respectueux de la volonté de l'auteur.

Figures

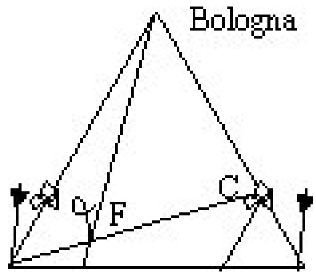


Fig. 1

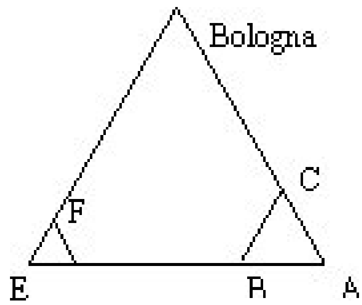


Fig. 2

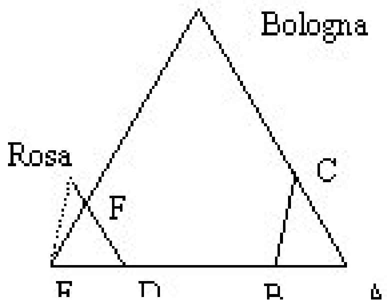


Fig. 3

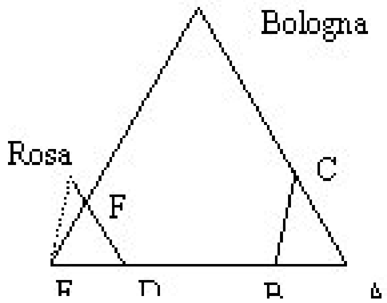


Fig. 4

Figures illustrant le problème XVIIe (sur la mesure des grandes distances) des Ex ludis rerum mathematicarum respectivement dans les mss. Typ. 422/2 de la Harvard College Library, f.13r [fig. 1] et Riccardianus 2110, f. 43v [fig. 2], ainsi que dans LEON BATTISTA ALBERTI, Opere volgari, a cura di Cecil Grayson, vol. III: Trattati d'arte, Ludi rerum mathematicarum, Grammatica della lingua toscana, Opuscoli amatori, Lettere, Bari, Laterza, 1973, p. 169 [fig. 3] et dans FRANCESCO FURLAN & PIERRE SOUFFRIN, Philologie et histoire des sciences: Le problème XVIIe des Ludi rerum mathematicarum, in "Albertiana", IV, 2001, pp. 3-20: 17 - par la suite dans FRANCESCO FURLAN, Studia albertiana: Lectures et lecteurs de L.B. Alberti, Paris, Librarie Philosophique J. Vrin & Torino, Nino Aragno Editore, 2003, p. 217-233: 230 [fig. 4].

B.1.3 K. Saito, *The diagrams in Codex P of Euclid's "Elements"*

General Principles for the Diagrams

The diagrams in manuscripts, which seem to be drawn in an awfully incorrect manner, follow some specific principles, though these principles are not so rigid. This is pointed out first by Netz, in his book of 1999, *The Shaping of Deduction in Greek Mathematics*. In my communication, I present some peculiar feature of the diagrams, and try to find underlying principles.

The following result is based on my preliminary research in the first four books of the *Elements*, based almost only one manuscript, that is, codex P (Vat. Gr. 190). However, there is no sign that we should assume that other medieval codices have very different principles about the diagrams.

The most conspicuous principles are the following:

1. The diagrams are "standardized":

   (a) an angle becomes a right angle if it can be: —though it can be otherwise – e.g., line ΔΓ in III-16;

   (b) two lines are drawn equal if they can be equal: e.g., in I-47, the rectangular triangle becomes rectangular and isosceles; in II-1, BD=DE=EΓ in the diagram;

   (c) a segment of a circle becomes a semicircle if it can be: e.g., III-24.

   Thus, there appear more squares, rectangles, isosceles and equilateral triangles in the diagrams of manuscripts than in their texts. Heiberg "generalized" these diagrams, for example, making two lines different if they are not always equal, followed by all the current translations. However, the "generalization" seems to be a recent tendency, for Barrow's edition preserves the standardized diagrams fairly well.

2. Metrical correctness is not so important.

   (a) Less important part of the diagram is disregarded:
   e.g., in II-5, the line AΓ, the left-half of the bisected line AB, is drawn much shorter than BΓ (right-half), because (I believe) the argument is developped mainly in the square constructed on BΓ.

   (b) Even the distinction between straight lines and curved lines is not of absolute importance:

   In IV-16, in a diagram in the margin, the sides of fifteen-angled figure are represented by concave arcs, so that they do not coincide with the circumference of the circle in which the figure is inscribed. This is also the case in some Archimedean codices, as reproduced in Netz' translation of the *Sphere and Cylinder*.

There are also some cases which seem to contradict the principle of "standardization". The square in II-4 is oblong. A possible explanation is that the scribe did not want to leave too much space around the diagram—a sort of *horror vacui*.

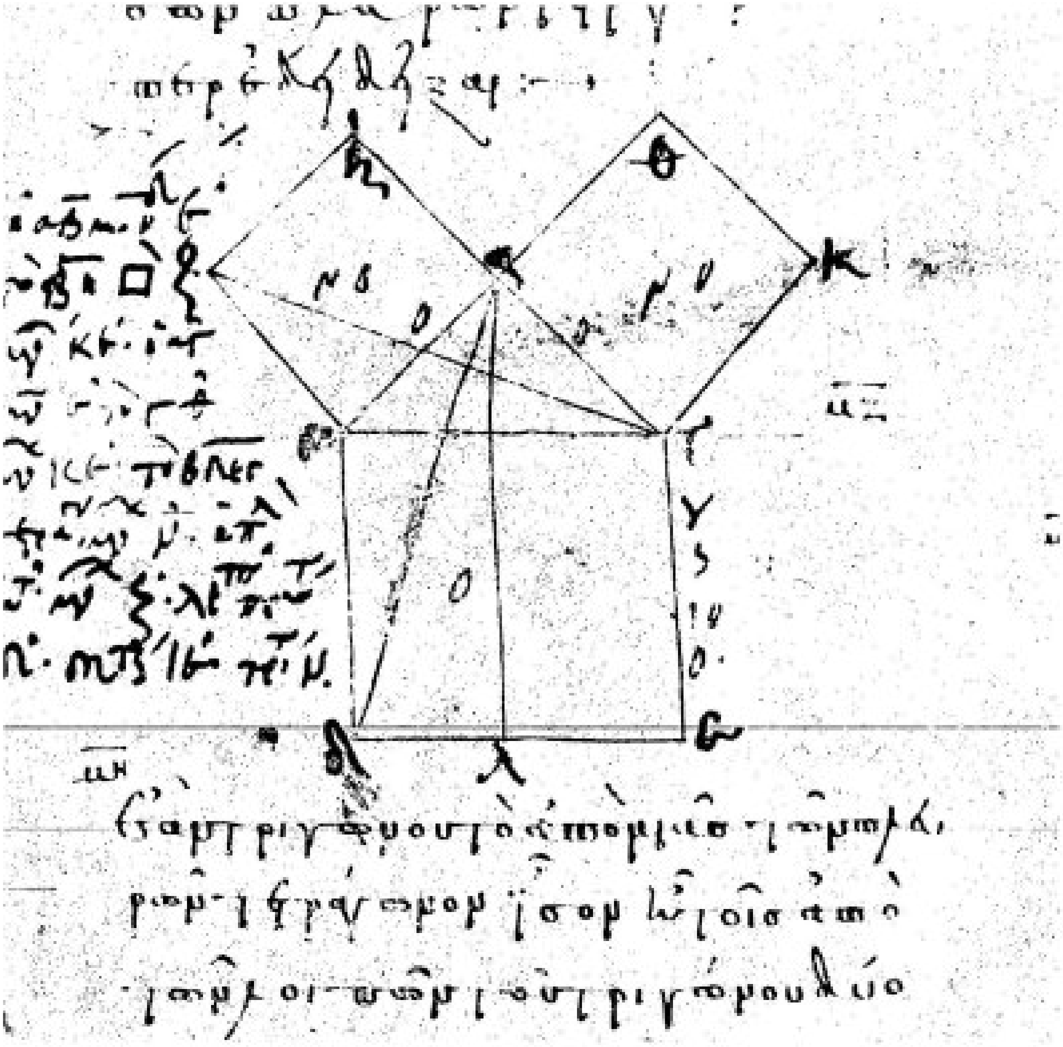
An Intriguing Case: *Elements* III-25

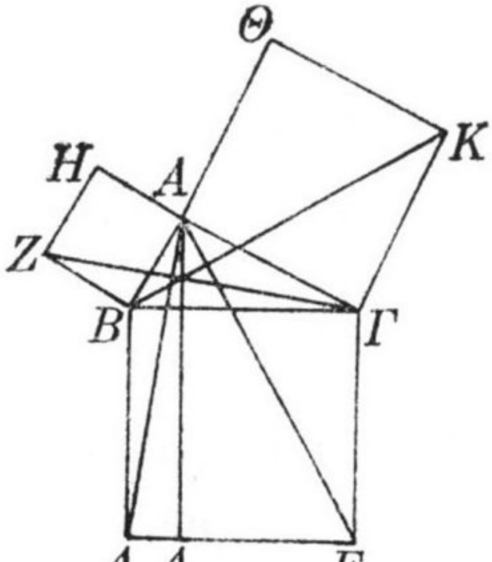
The diagrams for the proposition III-25 (to complete a given segment of a circle) present an interesting problem. We are accustomed to the three diagrams in Heiberg's edition (followed by all the translations, of course), which correspond to the three cases in the proof: the given segment being less, equal, greater than a semicircle. However, the manuscripts presents only one diagram in the column where the text and the diagram appear, and the three diagrams according to three cases appear in margin.

What is intriguing in the diagram in the column (not in margin) is that though it describes the given segment of circle as a semicircle, it marks two other points E and Θ, one in the semicircle and the other out of it, so that the diagram seems to serve for all the three cases. Other diagrams in the margin are likely to have been added afterwards, according to the three cases in the text.

We cannot exclude the possibility that the division of the proof into three cases is a result of later intervention, and the original proposition presented the proof only once in the text, accompanied by one figure applicable to three cases (another manuscript, codex B, presents one in the column, and two in the margin. In fact, the difference of the cases, which is apparent in the diagram, is not important in the text. The same procedure is repeated three times, and if one is indifferent to the place where the center of the circle falls (whether in the segment ABΓ, or on the chord AΓ, or out of the segment), one has no need of distinguishing the three cases.

It would be too hasty to try to give some conclusion to this problem, for the tradition of Book III is very complicated, accompanied by alternative proofs of several propositions.

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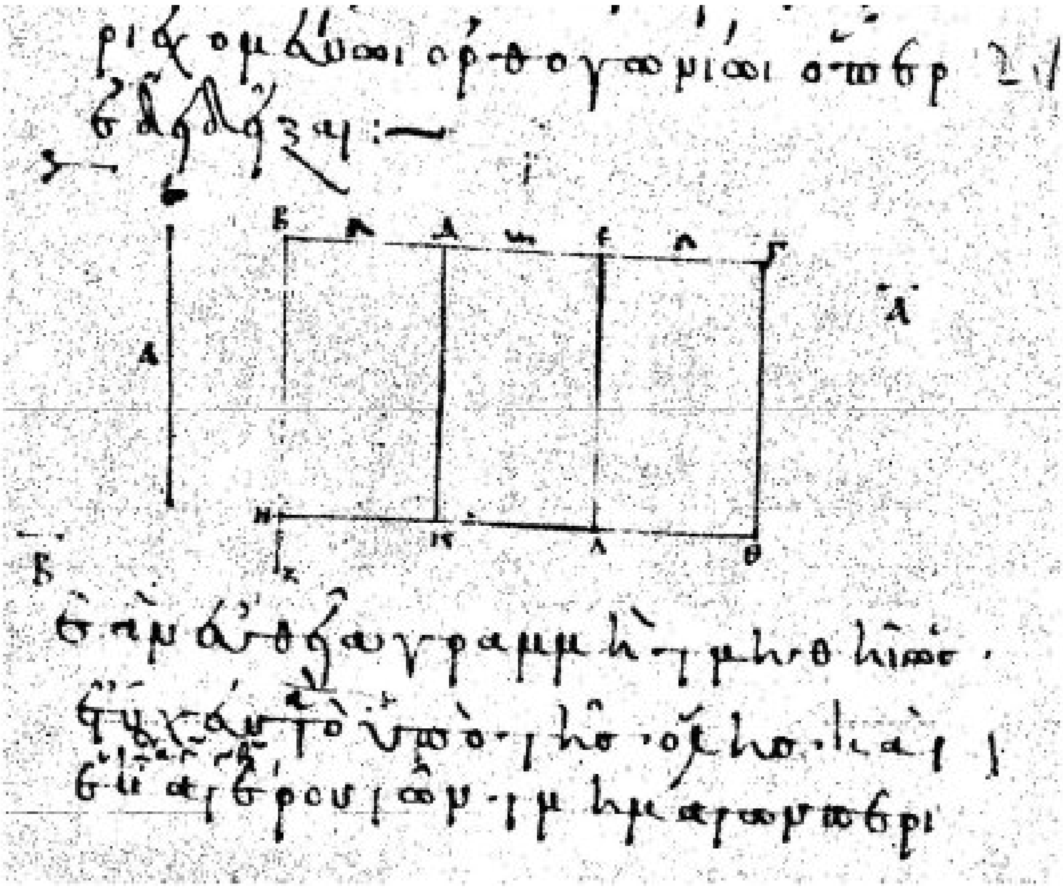

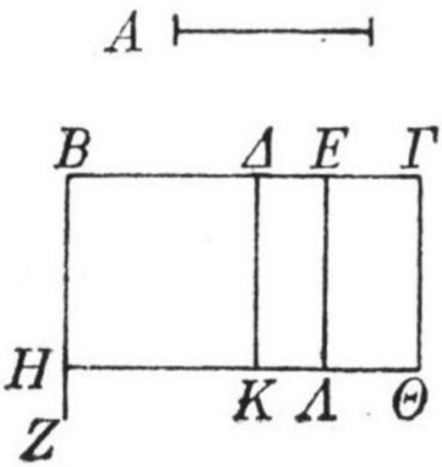
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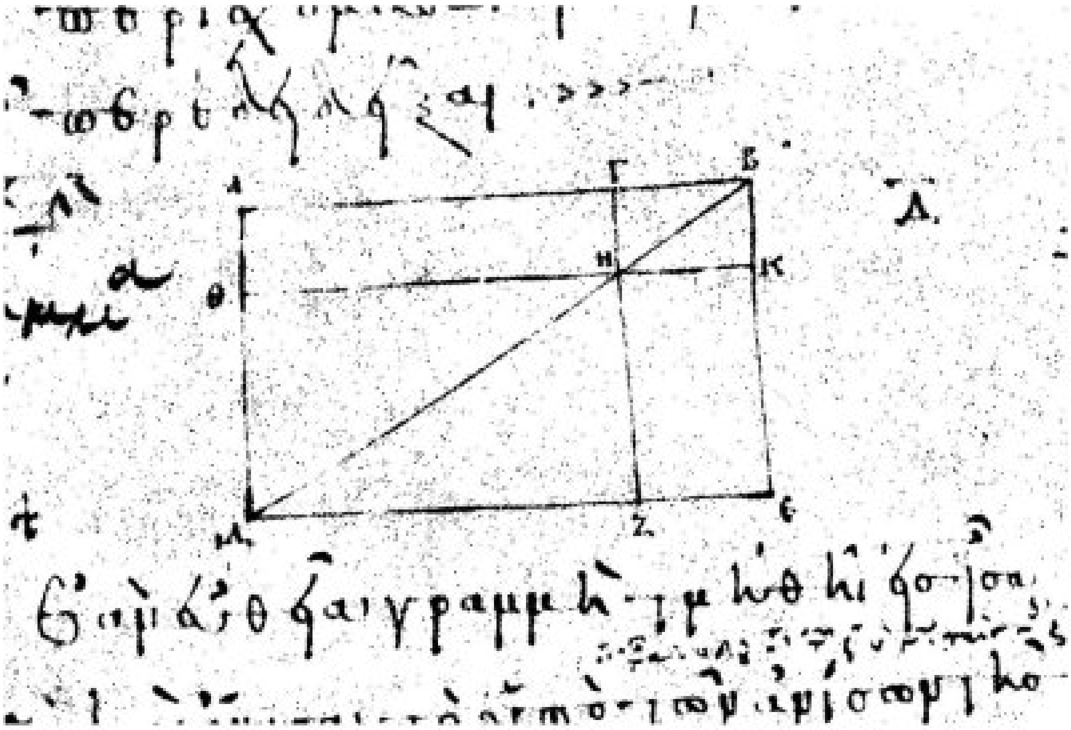

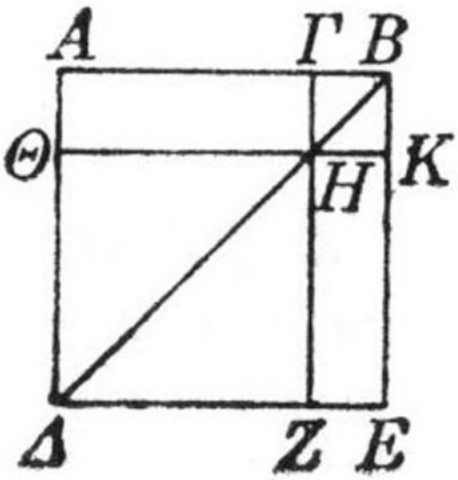
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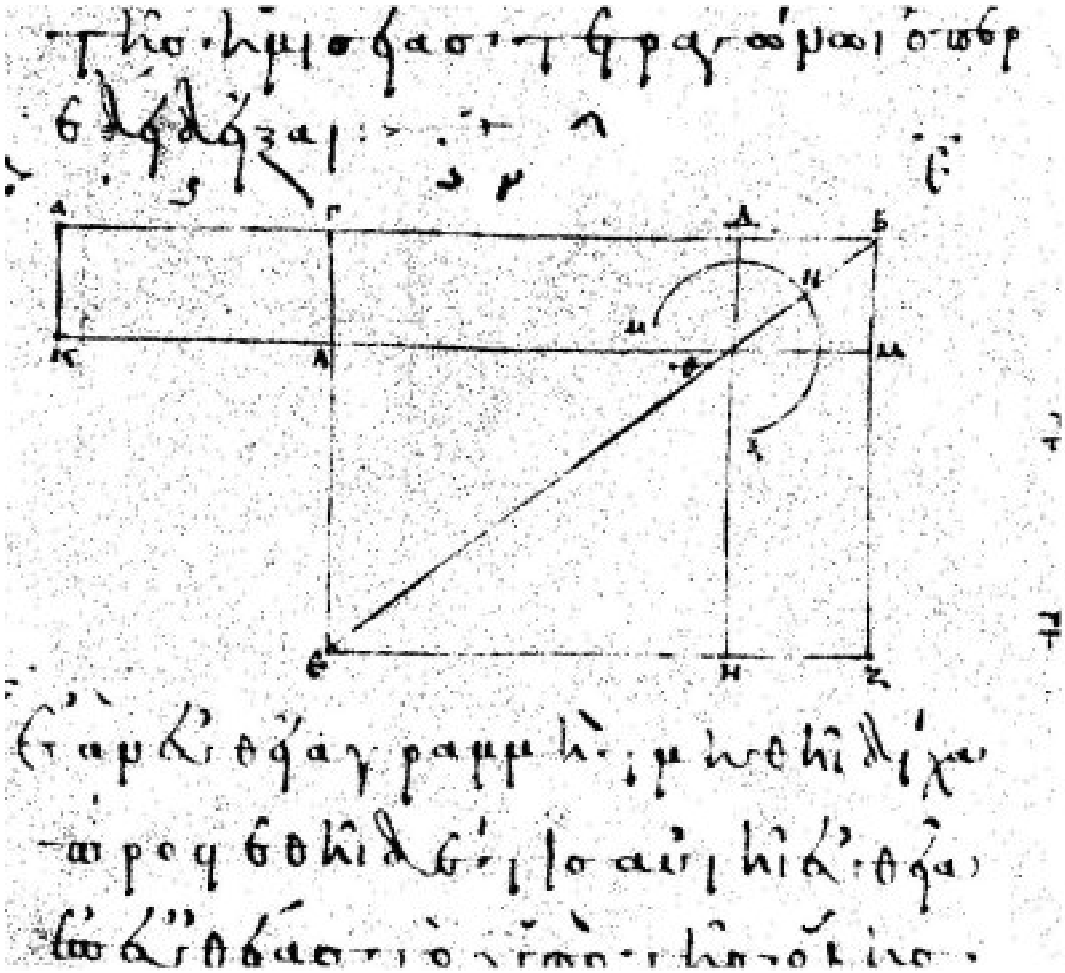

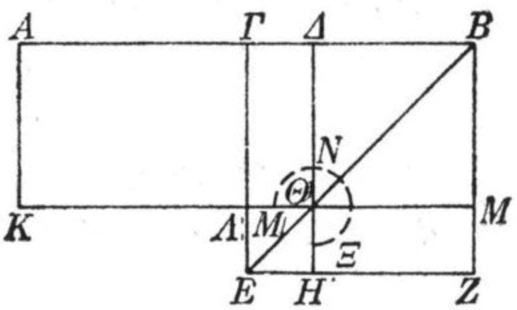
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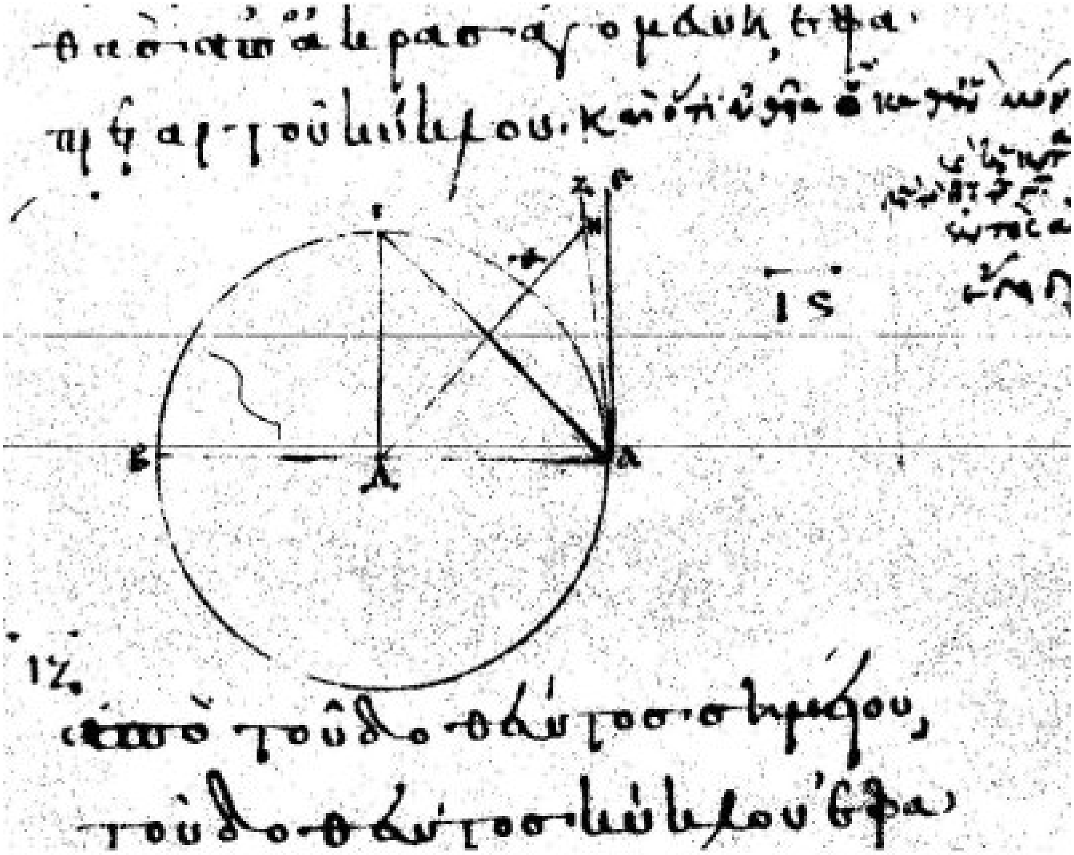

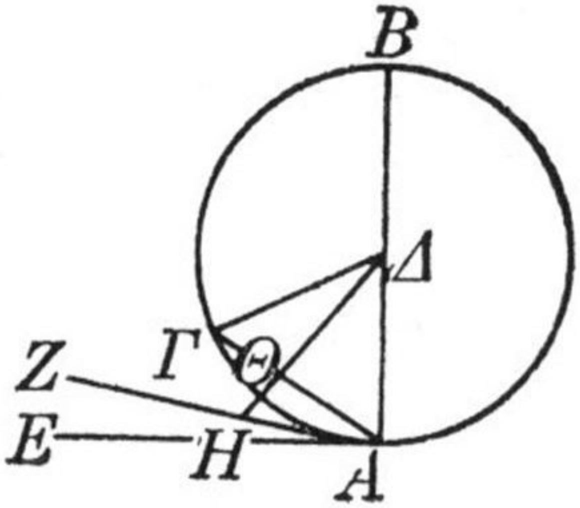
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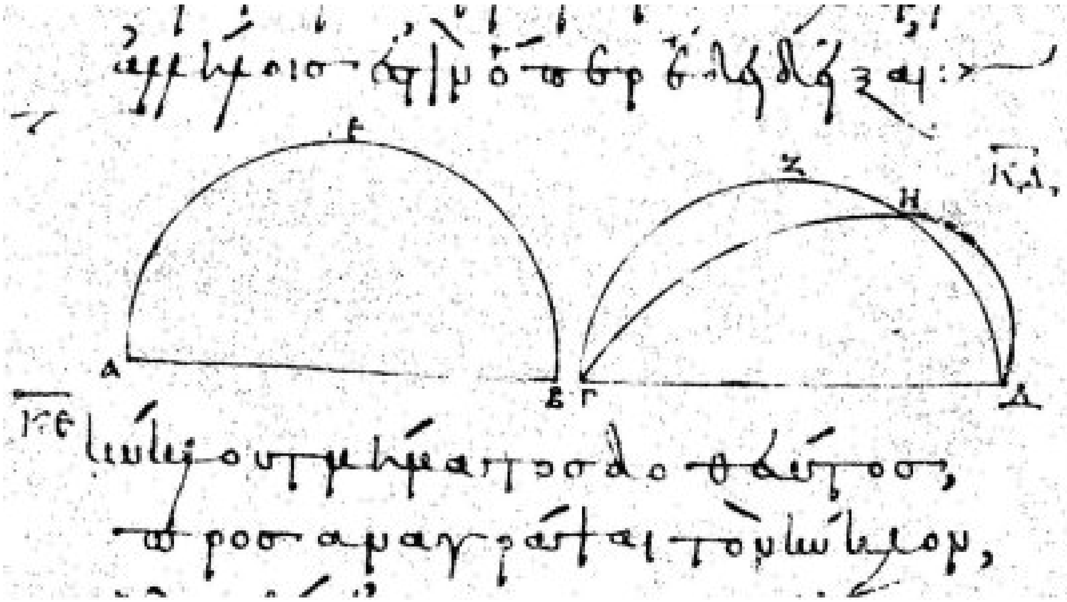

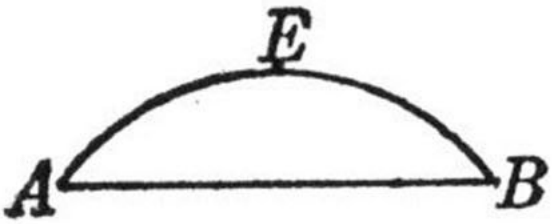
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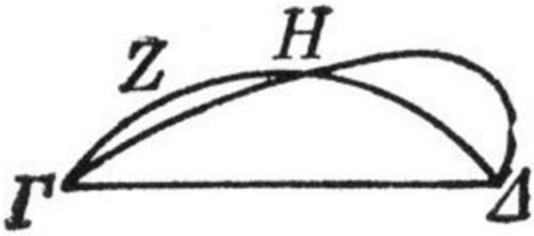
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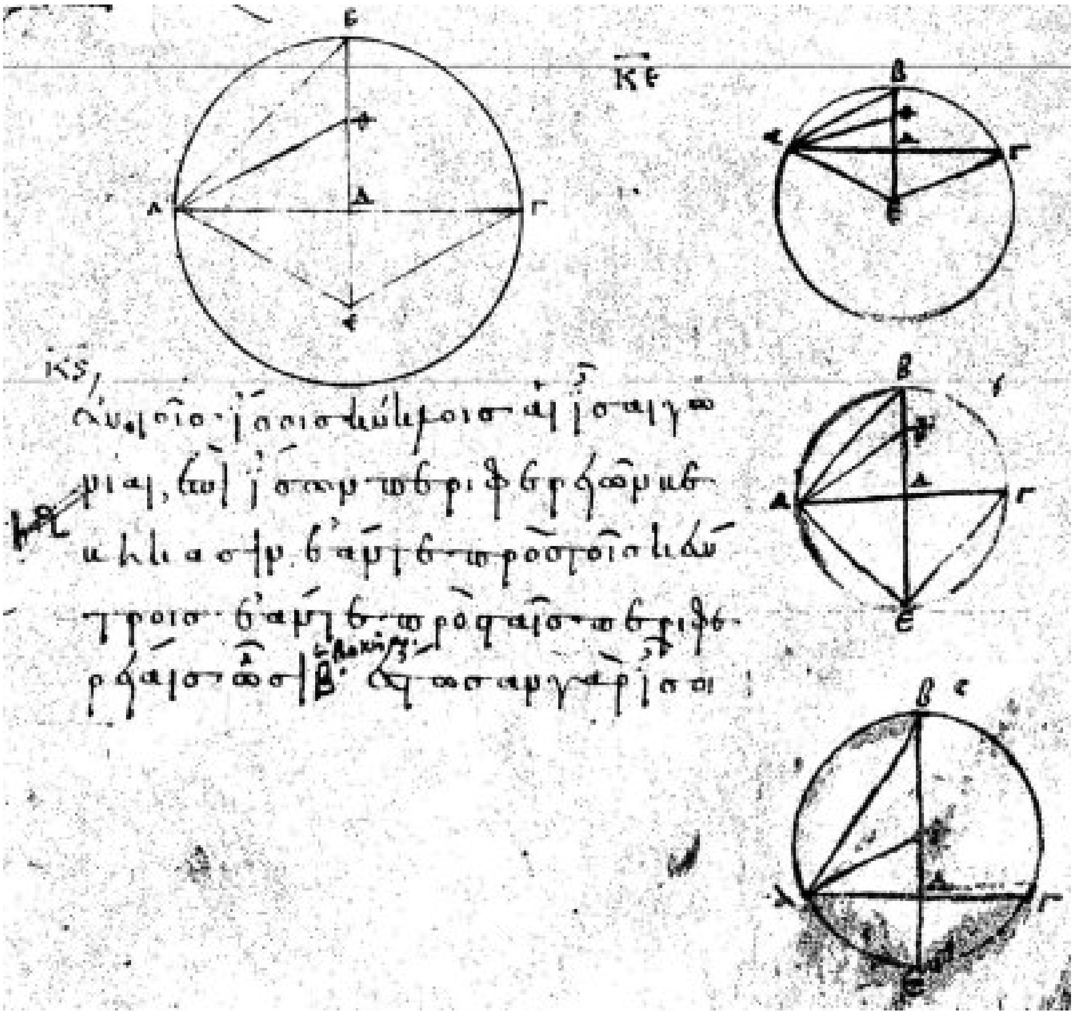

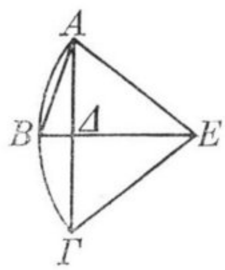
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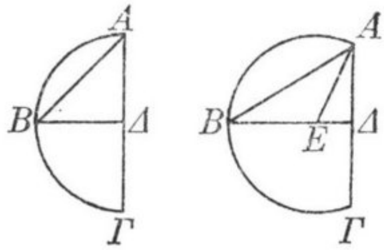
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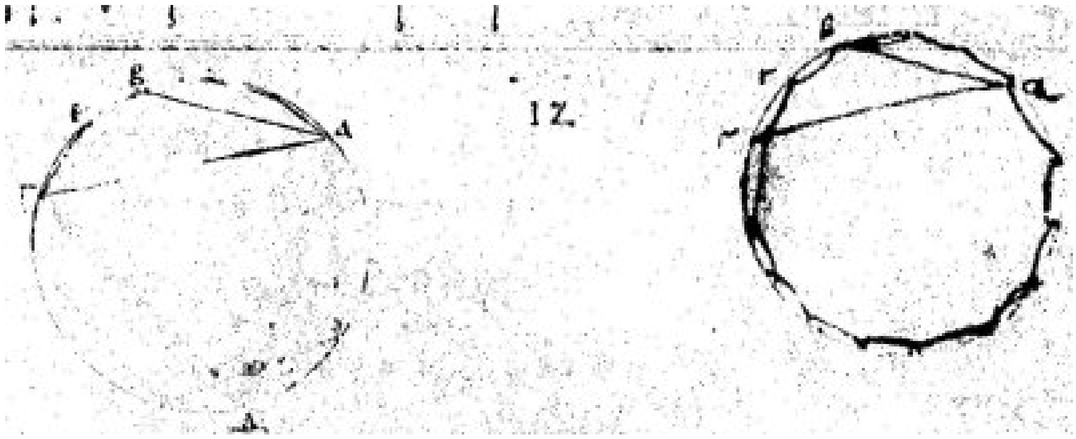
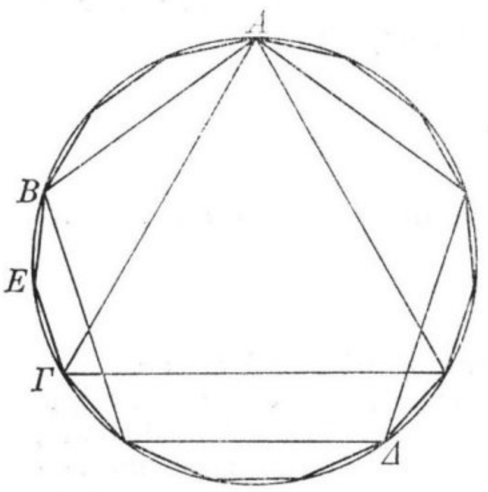




B.1.4 A. Sorci, *The case XIII.16 of Euclid's "Elements". Some remarks on the iconographical traditions and their editions*

In 1888 J. L. Heiberg completed the critical edition of the Greek text of Euclid's *Elements*. Heiberg's edition was taken as the definitive text of Euclid's *Elements* and after its publication most of the translations of the *Elements* were based upon it. Some translators were faithful to the original Greek source, whereas others preferred doing quite free translations to render the text clear enough for a modern reader. However, all equally explained their editorial criteria in the introductions, therefore the reader is acquainted with the features of translation he has in hand. The same doesn't hold true for *Elements*' diagrams.

Even though all translations are based upon the identical Greek text, they often show, especially in stereometric books, different diagrams for representing the same proposition. Moreover, none of them makes the slightest remark about the figures that accompany the text. Still less they point out the graphical rules and critical principles according to which the figures have been executed. The problem XIII.16 and its diagrams can be fit for inspecting this kind of editorial choices and for highlighting the differences between the modern and the 16th century editions of Euclid's *Elements*.

This proposition, concerning the construction of an icosahedron and its inscription in a given sphere, is an exemplary case of inquiry owing to geometrical complexity of the polyhedral construction and diversity of the drawing solutions employed by the editors. In Heiberg's edition the diagram outwardly seems to correspond with the drawings of manuscript tradition, but it can be noticed some interpolations and graphical conventions that may not be found in Greek codices. Heiberg passes over these interventions in silence and he only limits himself to indicating some variant drawings recovered in manuscripts.

Heiberg's silence has led many translators to regard the diagrams of his critical edition as accurate copies of the original drawings. As to proposition XIII.16, Thomas Heath and then Federigo Enriques, following the example of Heath, reproduce the same figure as Heiberg's edition, yet both add another figure in the mathematical commentary on the problem. Both Heath and Enriques maintain that they have drawn another figure because Euclid's figure wasn't clear enough. As a matter of fact, the diagram ascribed to Euclid corresponds with that in Heiberg's edition and not with those of Greek codices; but these claims suggest that the drawings of Heiberg's edition have been considered as faithful copies of the original ones. There are editorial choices even more radical than those adopted by Heath and Enriques.

Attilio Frajese, who published the Italian translation of the *Elements* in 1970, replaces Heiberg's diagram with another three-dimensional figure so as to delete any trace of the ancient iconographical tradition. Although the modern editions of the *Elements* have followed different criteria in representing geometrical figures, they all seem to be grounded on common assumption and to lead to similar consequences. They seem to share the tacit opinion that the figures are merely subordinate to the text. For this reason, the editors have deemed it legitimate to alter the drawings pertaining to the text without giving an account of their choices. It follows that we still rely on imperfect editions of the *Elements*, because their graphical corpus is somewhat or even totally different from the original one.

Contrary to what occurs in the modern translations, the 15th and the 16th centuries editions of the *Elements* show iconographical apparatus that tend to remain unaltered and faithful to the former manuscript tradition. We can separate the Renaissance printed Euclid into two main groups: the one refers to the Medieval Arabic-Latin redaction of Campanus of Novara and it includes the first printed edition, published by Erhard Ratdolt at Venice in 1482, but also the revision published by Luca Pacioli at Venice in 1509. The other numbers the Greek-Latin translation of Bartolomeo Zamberti, brought out at Venice in 1505, and the editio princeps of the Greek text by Simon Grynaeus at Basel in 1533.

Campanus' and Zamberti's versions gained a great success, so much so that they were repeatedly reissued in conjunction during the 16th century. First they came out at Paris in 1516 under the imprint of Henri Estienne I, then they were again published by Johann Herwagen at Basel in 1537, 1546 and 1558. Besides these integral printed versions of the *Elements*, we must mention several partial or mixed editions that are based upon one or both Campanus' and Zamberti's text. In 1536 Oronce Fine printed a partial edition in which the Greek enunciations of the propositions alternated with the Latin text of Zamberti's translation. Jacques Peletier and Franciscus Flussates, Comte de Candale, used Campanus' version as well as Zamberti's for arranging their Latin editions (Lyon, 1557 and Paris, 1566 respectively) and Niccolò Tartaglia did the same with regard to his Italian translation of the *Elements* (Venice, 1543). It can be noticed that Campanus' redaction of the *Elements* differs from Zamberti's translation not only in lexical choices, in order and number of propositions, but also in characteristic and rigidly definite set of diagrams. Actually, from a superficial examination it appears that Campanus' redaction is accompanied with a graphic corpus peculiar to it, but distinct from the one of Zamberti's translation.

If, on one hand, Ratdolt's edition shows a collection of diagrams quite similar to the ones that usually complete the manuscript versions of Campanus, on the other, the diagrams in Zamberti's translation correspond with those of Greek codices. So it wouldn't be wrong to claim that each of two editions can be singled out for its text as well as for its illustrations. These textual and iconographic features are also preserved in the following editions of *Elements*. The editors stick to their sources and reproduce them in texts as well as in figures faithfully, except for secondary adjustments. Thanks to such editorial choices, the two sets of illustrations are handed on unaltered in their qualities and reciprocal diversities until Federico Commandino breaks up this tradition with his Latin and Italian translations of the *Elements* in 1572 and 1575.

The manuscript tradition seems too wide to be examined in this circumstance, while the printed editions present a corpus of figures which fits our purpose better on account of its greater uniformity and regularity. Therefore I shall confine myself to the printed versions, only focusing my attention on the main complete editions of the *Elements*. The problem XIII.16 can exemplify the graphical differences between the two traditions, but also some common traits. Actually, all editions represent the geometrical figures schematically without recourse to any device for achieving pictorial or three-dimensional effects. This practice doesn't raise difficulties as for the illustration of plane geometry, but it can prove unsuitable to solid geometry.

Often the solids, which accompany the theorems, look flat and visually illegible, as if they are squashed against the pages. Although geometrical drawings don't obey any explicitly formulated rule, they display a uniformity that seems to proceed from some consistent and thought-out convention. The permanence of such graphical formulae leads to think geometrical diagram conforms to logic differing from the one that rules other sorts of image. Actually, geometrical drawing doesn't mean to achieve effects of visual illusionism; rather, it helps reader's mind to grasp what the text cannot explain by itself. For this reason, the diagrams of geometry, especially those of the *Elements*, form an inseparable unity with the text of propositions. More exactly, the diagram shows what is established by the constructions of theorems. The figures of the *Elements* don't represent the enunciations of propositions, namely either what is given, or what is sought; on the contrary they display what is requested by construction for concluding the proof. The procedure adopted to execute this kind of representation consists in orderly and constant series of operations.

Although that procedure doesn't stem from a codified canon, it might be called "sequential", because it outlines each figure and its components by sticking to the sequence of construction. In fact, the first step of construction is also the first element of diagram to be portrayed; it is usually parallel to the plane surface of page and it exhibits its own shape without any distortion. The parts, which gradually are added to the first, get to the point of overlapping each other, so they are forced to bend slantwise towards the background of page. The visual effect will be unsatisfactory and not convincing at all, but, on the other hand, the adherence to the text will prove almost complete, because the diagram will reflect not only the content, but also the order of geometric construction faithfully. Actually, in geometrical diagrams all steps of construction are shown one on top of the other as well as in the relative text they are introduced one after the other. The practice will become plainer by examining the problem XIII.16 of the *Elements*. In this proposition, Euclid teaches how to construct an icosahedron in a sphere and then he demonstrates that the side of the icosahedron, compared to the diameter of circumscribed sphere, is the irrational straight line called minor.

The icosahedron, together with tetrahedron, octahedron, cube and dodecahedron, is one of the five regular polyhedra, namely the convex solids with equal regular faces that can be inscribed in a sphere. The icosahedron, which consists of twenty equilateral triangles, is a very complex solid and certainly one of the most difficult to draw. However it constitutes a typical case of the way of representing the geometrical figures and therefore it deserves a close inspection. Both Zamberti's and Campanus' editions presents a couple of diagrams in accordance with the text of *Elements*. As a matter of fact, Euclid arranges the construction of tetrahedron, octahedron, cube and icosahedron in two distinct stages. At first he considers a semicircle with diameter equal to the diameter of the given sphere. This preliminary operation enables him to find the parameters required for proceeding to production and inscription of polyhedron in the sphere. As established by Euclid, Zamberti and Campanus outline a semicircle on which it's determined the ratio between the edge of polyhedron and the diameter of sphere. Then, in a separate diagram, it's represented the construction of icosahedron. Really, none of the two editions shows the image of the whole solid, but only of the part described in the construction of problem. The final results look dissimilar, yet both follow the same procedure of representation.

Zamberti's drawing, as well as the corresponding one in Greek edition, reproduces all steps of Euclidean construction according to same order. At first two regular pentagons and a regular decagon must be inscribed in a circle of radius equal to the chord of the semicircle previously described. These initial elements are drawn parallel to the plane surface of page, so they don't undergo any alteration in the shape of figures. The two regular pentagons, which overlap inversely, keep the equality of sides and the radius of the circle is also equal to the chord of the semicircle previously described. Once this operation had been executed, it must set up perpendiculars from the vertices of one of the two pentagons to the plane of the circle; then the extremities of these perpendiculars, which are equal to the radius of circle, are joined so as to complete another regular pentagon. After that each vertex of this pentagon is joined to the two nearest vertices in the other pentagon. Thus it is obtained ten equilateral triangles that form the medial ring of the icosahedron. Instead of expanding in the depth of sphere, this series of triangles is arranged round the circle in such a way as to be shown coplanar with the circle and not perpendicular to its plane. The diagram doesn't present any device for bringing the figure into relief; on the contrary it flattens all parts of polyhedron against the surface of page.

Finally Euclid finds the poles of the polyhedron and only traces two of the ten remaining equilateral triangles, by joining each pole to two angular points of the corresponding pentagon. The poles, which would be diametrically opposed and would lie the one over and the other under the pentagons, were instead folded back within the outlines of pentagons. As a consequence, the upper triangle and the lower one appear levelled to the same plane of pentagons. The graphic conventions of Campanus' edition don't diverge basically from the ones of Greek-Latin versions of the *Elements*, even though the scheme of icosahedron is outwardly rather unlike. Truly the two figures agree in the first part of construction, because Campanus' diagram exhibits the circular section of the sphere with the decagon and the two inscribed pentagons. Instead it omits all the remaining components of the solid and adds an element that is missing from Zamberti's edition, namely the generator semicircle of the given sphere. Therefore Campanus' diagram is merely defective, but not radically different from Zamberti's. Both Zamberti's and Campanus' diagrams look incomplete and not at all clear, yet both result from a settled consistent procedure. This sole comparison doesn't suffice to define the criteria the editions of *Elements* should follow in establishing the corpus of geometrical diagrams. Nevertheless, it shows that geometrical figures require a critical analysis similar to that usually devoted to the text so as not to delete the history and distort the meaning of the text itself.

B.1.5 M. Malpangotto, *Some remarks about the diagrams of "Theodosii Sphaericorum libri tres"*

Theodosii Tripolitae Sphaericorum libri tres were written during the 1st century b. C. by Theodosius, mathematician and astronomer living in Tripoli on Fenicia coast. In these three books he gave a logically structured explanation to properties of circles and arcs lying on sphere surface, arranging a series of proposition destined to become the theoretical foundation of astronomy and completing on this subject the few notions in Euclid's *Elements*.

Theodosius' work followed the same traditio shared by the main Greek mathematical classics. After an early circulation in its original language, during the 9th century it was translated into Arabic. It is from Arabic redactions that this work was translated into Latin to nourish Latin West learning, since twelfth century. All these passages modified, enriched and corrected Spherics text. In spite of all textual changes -sometimes heavy-, the whole manuscript tradition -Greek, Arabic, Latin- kept the original Theodosius' diagrams unchanged. And the same happens in the first printed editions: Venice 1518 and Vienna 1529, both presenting an Arabo-Latin version; Paris 1558, being the Greek editio princeps followed by an accurate translation.

In 1558 Francesco Maurolico published at Messina Theodosii Sphaericorum elementorum libri tres ex traditione Maurolyci. He gets into the Arabo-Latin tradition of Spherics to carry out an original version, "ex traditione Maurolyci" properly, so expressing a deep change in intending sphaerica. The perspective changes: now sphere comes into the foreground and it is on its surface that our mathematician prefers to articulate his reasoning, while all previous versions made reference to planes, whose intersections with sphere give origin to circles involved by Theodosius' propositions. Diagrams in Theodosii Sphaericorum libri III ex traditione Maurolyci reflect exactly this new approach: circles and arcs involved by propositions always are placed on the sphere, which is depicted as a whole, in a tridimensional and prospectic way. According to this, figures pertinent to reductio ad absurdum are not drawn on sphere: these proofs considering situations impossible to come true in a spherical context. Among Maurolico's coherent and rational choices characterizing all the 84 propositions, 9 diagrams seem to contrast with the usual logical rigour. Their unusual shape isn't due to particularly hard troubles of representation: some of them are so easy to be drawn, that it would be enough only one circle added, to conform them to the rest of the work. Neither it rises from lacking care nor from chance choices, as, for example, two of these diagrams support propositions -even meaningful- which are Maurolico's additions to the Arabo-Latin corpus of Theodosius' Spherics. Logical rigour about geometrical figures and exclusion of the above-mentioned simple reasons, urge a careful study on these propositions, in order to understand an eventual common motive or different choice criteria applied to every single diagram. Nowadays these nine cases represent an editorial problem, compelling editors to choose between the following solutions: to redraw them faithfully, in their original form contrasting with the general ratio of the three books, or to modify them, forcing -an eventual- Maurolico's will. At last in Theodosii Sphaericorum elementorum libri tres ex traditione Maurolyci there are other interesting cases, although less uncertain for editors. All needed emendation to these diagrams must be made by adding lacking elements in order to make them coherent with the demonstrations they refer to, without altering their form.

Looking for possible solution to problems here briefly accounted, it will be meaningful to consider also editorial choices adopted in the first edition of Theodosii Sphaericorum Libri III after Maurolico's one. It was edited in 1586 by Christophorus Clavius, who states reasons for his choices about diagrams as follows: "Figuras quoque que in Graeco exemplare extent, plerumque neglegimus, quod ille quas Maurolicus pinxit, commodiores sint et ad indelligendas res sphaericas multo faciliores." (Praefatio of Theodosii Tripolitae Sphaericorum Libri III. Romae, 1586), attesting the greater clearness of Maurolico' geometrical figures and opening their editorial fortune.

Chief bibliography:

Georgii Vallae...De expetendis et fugiendis rebus opus ... Venetiis, in aedibus Aldi Romani

impensa, ac studio Ioannis Petri Vallae filii pietiss. mense Decembri 1501

Sphaera cum commentis in hoc volumine contentis, videlicet: Cichi Esculani cum textu. Expositio Joannis Baptiste Capuani in eandem. ... Theodosii de Spheris. ... Venetiis. Impensa heredum quondam Domini Octaviani Scoti Modoetiensis ac sociorum. 19 Januarii 1518

Theodosii de Sphaericis libri tres, a Joanne Voegelin Hailpronnensi, ... restituti et Scholiis non improbandis illustrati. Viennae, in officina Joannis Singrenii. Anno 1529. 18 Martii.

Theodosii sphaericorum elementorum libri III, ex traditione Maurolyci Messanensis mathematici;.... Messanae, in freto Siculo, impr. Petrus Spira mense Augusto 1558

Theodosii Tripolitae Sphaericorum libri tres, nunquam antehac graece excusi. Iidem latine redditi per Joannem Penam ... Parisiis, apud Andream Wechelum, sub Pegaso, in vico Bellovaco. Anno salutis 1558

Theodosii Tripolitae Sphaericorum Libri III, a Christophoro Clavio Bambergensi Societatis Jesu perspicuis demonstrationibus ac scholiis illustrati. ... Romae, ex typographia Dominici Basae. 1586

Theodosius Tripolites Sphaerica von J. L. Heiberg Berlin Weidmannische Buchandlung 1927

B.1.6 P. Crozet, *Editer les figures des manuscrits arabes des géométrie: l'exemple d'al Sijzî*

Introduction

Parmi les difficultés auxquelles se heurte l'éditeur de traités mathématiques arabes dans le traitement des figures géométriques, beaucoup tiennent à l'éloignement dans le temps des traditions scientifiques dont elles sont issues et, partant, sont souvent liées à l'histoire de la transmission des textes. La liste serait longue, en effet, de tous les avatars susceptibles de subvenir aux figures lors de la copie des manuscrits : celles ci peuvent être par exemple totalement omises, remplacées par des espaces destinés à être ultérieurement remplis mais qui ne l'ont jamais été, ou encore n'être recopiées que partiellement ; elles peuvent également être reléguées à la fin du texte ; elles peuvent encore être retournées ou inversées ; des points peuvent perdre leur dénomination, d'autres peuvent être confondus, des lignes peuvent être ajoutées ou retranchées, l'allure générale peut être considérablement modifiée, etc. Or si ces représentations sont bien entendu un auxiliaire indispensable à la pensée du mathématicien, l'éditeur scientifique d'aujourd'hui peut-il passer outre la tâche de livrer, autant que faire se peut, des figures semblables à celles tracées par l'auteur lui même ? Une telle exigence se heurte en réalité à un certain nombre de problèmes dont la plupart restent encore à résoudre du fait de l'absence de normes rigoureuses en matière de restitution de figures.

L'apparat critique des meilleures éditions est du reste souvent muet sur ce point, la restitution de la pensée mathématique, non sans raison, occupant plus volontiers l'attention que ce qui lui a pu lui servir de support. Que faire donc des figures qui, pour une raison ou pour une autre, apparaissent comme "erronées", incomplètes, ou peu en accord avec le fond ou la lettre du texte qu'elles accompagnent ? Doit-on reporter ou gommer les particularités des figures ? Comment traiter les figures dont la transmission manuscrite a laissé des reproductions multiples aux allures dissemblables ? Bien entendu, comme pour le texte lui-même, il appartient à l'éditeur de faire un choix, de prendre des options. Celui ci ne saurait en outre se contenter de donner des fac-similés des figures présentes sur les manuscrits disponibles : outre le fait qu'un tel projet deviendrait rapidement irréalisable dans le cas de manuscrits multiples, le rôle d'un éditeur est bien de proposer un ensemble - texte et figures - qui puisse servir de référence, et non de laisser le lecteur aux prises avec une analyse codicologique, historique, linguistique et scientifique qui resterait à mener sur les différentes versions en présence. Dans une telle matière, les figures doivent sans doute, au même titre que le texte à proprement parler, faire l'objet des soins de l'éditeur et nécessiter un traitement similaire. Au même titre que le texte encore, et comme nous le verrons plus loin, elles devraient pouvoir aider à la reconstitution de la chaîne des manuscrits et indiquer des filiations. J'aborderai ici un certain nombre de ces difficultés en prenant quelques exemples tirés de l'oeuvre géométrique d'un mathématicien persan (mais écrivant en arabe) du dernier tiers du Xe siècle, Ahmad ibn Muhammad ibn ‘Abd al Galil al Sigzi.

Generalite des figures

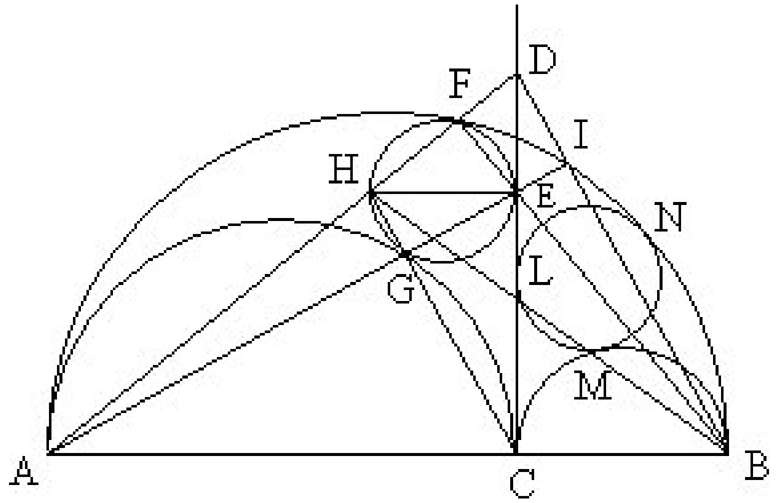
Avant toutefois d'examiner les manuscrits eux-mêmes, je voudrais soulever au préalable un problème d'un ordre plus général qui est lié à la façon dont la figure est faite et, par là, transmise. Il s'agit de ce que j'appellerai le problème de la *généralité* de la figure, à savoir le rapport qui existe entre la figure effectivement tracée (*sura*) et le caractère général de la figure géométrique (*sakl*) qu'elle représente. Les normes tacitement admises de nos jours conduisent à éviter, par exemple, de représenter, dans un écrit, un triangle quelconque à l'aide d'un triangle équilatéral, ou un angle quelconque au moyen d'un angle droit. Rien ne permet de penser qu'il en ait été strictement de même à l'époque qui nous intéresse. En effet, les figures qui nous ont été transmises font en effet souvent apparaître des triangles équilatéraux en lieu et place de triangles scalènes, des carrés pour des parallélogrammes, des segments égaux pour des segments inégaux, etc., sans qu'une gêne particulière en soit produite pour la compréhension du lecteur. Ceci correspond il au choix du mathématicien, qui pourrait répondre alors à un souci esthétique ou de tout autre nature ? Est ce là, au contraire, le fruit du hasard, l'auteur n'ayant pas de réticence particulière à l'usage du particulier pour représenter le général ? Enfin, cette distorsion n'est elle pas plutôt le fait d'une distorsion induite par la transmission même des manuscrits ?

Une réponse définitive à cette question des normes serait sans doute de grande utilité pour les éditeurs de textes anciens ; elle nous apparaît néanmoins prématurée en l'état actuel des recherches. Je voudrais toutefois introduire ici un exemple qui semble suggérer que c'est bien chez le mathématicien lui même qu'il faille chercher des réponses, dans la mesure où cette relation du général au particulier est là en parfaite correspondance avec le texte lui même.

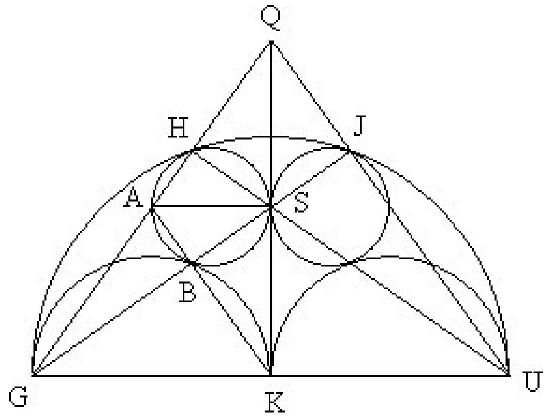
Cet exemple est celui de la cinquième proposition du Livre des lemmes attribué à Archimède, livre que commente al-Sigzi et pour lequel il donne un certain nombre de démonstrations alternatives (ce qui n'est d'ailleurs pas véritablement le cas pour cette proposition)[1]. On considère, sur le diamètre AB d'un demi-cercle, un point quelconque C ; on mène les demi cercles de diamètres AC et CB, et on élève en C la perpendiculaire CD à AB ; on construit enfin, de part et d'autre de cette perpendiculaire, les deux cercles tangents à la perpendiculaire et aux deux demi cercles. La cinquième proposition du Livre des lemmes assure que ces deux cercles sont égaux. La figure présentée par les éditions successives de l'ouvrage est similaire à la figure suivante, où les deux cercles égaux sont les cercles HGEF et LMN[2].

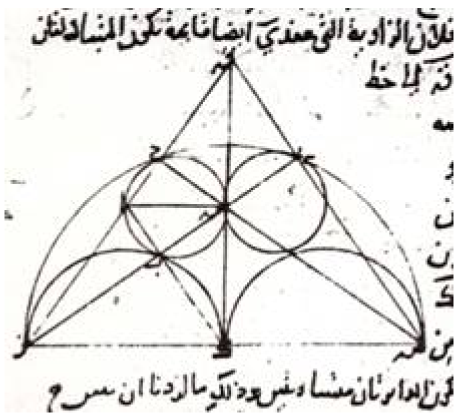
[1]Sigzi, Risala fi al gawab oan al masa'il al-lati su'ila fi hall al askal al-ma'kuxat min Kitab al ma'kuxat li-Arsimidis, MS 2458 (Bibliothèque Nationale, Paris), fol. 6r.

[2]Voir par exemple : Archimède, éd. Ch. Mugler, 4 vol., Les Belles Lettres, Paris, 1971, III, 141.



La figure présentée par al-Sigzi correspond par contre à la figure suivante, où le point qui partage le diamètre du grand demi cercle, ici le point K, est clairement au milieu de ce diamètre (ici le segment GU), alors même que le mathématicien l'avait explicitement déclaré quelconque quelques lignes plus haut :





Or ce cas particulier introduit une propriété supplémentaire qui n'existe pas dans le cas général, à savoir que les deux cercles tangents à la perpendiculaire KQ sont également tangents entre eux. De plus, et c'est là l'intérêt de l'exemple, le texte lui même s'accorde avec cette particularité. En effet, al-Sigzi parle de l'un des deux cercles comme étant de diamètre AS, où S est le point de tangence de ce cercle avec KQ, et introduit l'autre comme étant le cercle SJ, où J est l'intersection du prolongement de GS avec le demi cercle de diamètre GU. Or ce second cercle ne peut être ainsi dénommé que si K est le milieu du segment GU. Dès lors, l'éditeur du texte n'a d'autre choix que de respecter et le texte et la particularité de la figure.

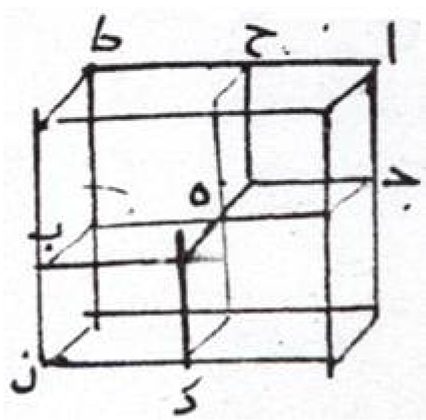
Cette particularité nuit-elle à la validité de la preuve ? Quoique le lecteur d'aujourd'hui puisse éventuellement être gêné pour appréhender l'énoncé de la proposition, la réponse ne peut être ici que négative. La démonstration d'al-Sigzi, semblable à celle qu'il attribue à Archimède, n'utilise en effet que des constructions mettant en jeu le cercle de diamètre AS et aboutit à l'égalité GK.KU = GU.AS ; la conclusion se borne alors à utiliser la remarque qu'un développement similaire pour le second cercle donnerait la même égalité, où AS serait toutefois remplacé par le diamètre de ce second cercle, ce qui permet de déduire l'égalité des deux diamètres.

Rien ne permet donc d'affirmer ici, bien au contraire si l'on en croit cet exemple, que normes et usages aient été de tout temps identiques, ou encore qu'ils aient toujours été appliqués avec une même rigueur. Une telle remarque ne facilite sans doute pas la tâche de celui qui tente aujourd'hui de restituer des figures dont on imagine volontiers qu'elles ont pu être largement déformées par la succession des copies manuscrites.

Cas d'un manuscrit unique

Si la multiplicité des manuscrits pose un certain nombre de problèmes sur lesquels nous allons revenir, le fait qu'une figure nous parvienne par le biais d'un manuscrit unique engendre parfois, lorsque cette figure se révèle " erronnée ", des difficultés que d'autres témoins auraient pu lever.

C'est le cas pour une figure tracée par al-Sigzi pour donner un équivalent géométrique de l'identité algébrique $(a + b)^3 - a^3 - b^3 = 3ab(a + b)$:

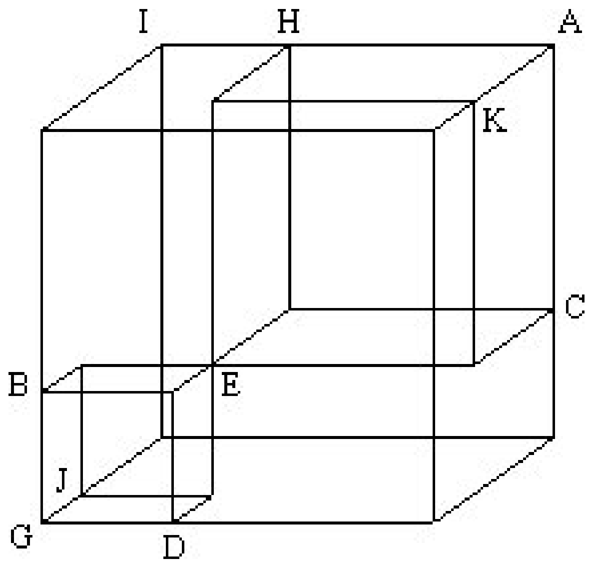


Cette figure est issue d'un manuscrit très vraisemblablement autographe - ce qui n'est pas pour diminuer les difficultés - le célèbre MS 2457 de la Bibliothèque Nationale (Paris), dans lequel al-Sigzi a copié, sans doute pour son usage personnel, un certain nombre de traités de ses contemporains et prédécesseurs, et aussi quelques-uns des siens propres, dont celui qui nous occupe[3]. Il s'attribue la copie de certains de ces textes, l'écriture étant identique, sauf pour les dernières pages du recueil.

On s'aperçoit rapidement que cette figure ne peut en aucun cas représenter un solide en trois dimensions, du moins de la façon qui nous est habituelle : le point (E) se trouvant sur la face arrière du petit cube inférieur, il se trouve alors, selon cette figure, sur la face arrière du grand cube; mais se trouvant sur la face avant du petit cube supérieur, il se trouve alors, toujours selon cette figure, sur la face avant du grand cube ; de sorte qu'il se trouverait à la fois sur la face avant et sur la face arrière du grand cube, ce qui est impossible.

La figure qui semble correspondre à l'intention du mathématicien est la suivante :

[3] *Livre sur la mesure des sphères par les sphères* (Kitab fi misaha al-ukar bi-l-ukar), MS BN 2457, fol.195v-198r ; voir notre édition, avec traduction et commentaires, dans : Pascal Crozet, " L'idée de dimension chez al-Sijzi ", *Arabic Sciences and Philosophy*, vol. 3 (1993), pp. 251-286.



En effet, si l'on ôte du grand cube AG les deux petits cubes AE et EG, il reste les trois solides égaux CD, HJ et KB, où nous introduisons deux points, J et K, qui n'apparaissent pas sur la figure. Le problème est que ces deux points ne sont pas dans le texte non plus, puisqu'al-Sigzi évoque les solides CD, HG et AB qui, écrit-il, sont tous les trois entourés par AI, HI et AH, ce qui est vrai pour CD, HJ et KB, mais non pour CD, HG et AB. Notre impression est qu'il s'agit ici d'une erreur lors de la copie du texte, les graphies arabes pour HG et HJ d'une part, et AB et KB d'autre part, étant assez proches.

Il reste néanmoins qu'une telle intervention, à la fois dans le texte et la figure, reste problématique. Se contenter d'avancer que l'auteur ait pu se copier lui même sans prendre beaucoup de soin à son travail de copiste ne peut que laisser la porte ouverte à toutes les dérives interprétatives. Dans une telle situation, il appartient alors sans aucun doute à l'éditeur soit de reproduire la figure et le texte tels qu'ils apparaissent sur le manuscrit, soit de proposer une interprétation dûment étayée en fournissant, dans l'apparat critique, sinon un fac similé de la figure du manuscrit, du moins une description précise de celle ci.

Multiplicite des manuscrits

Mais d'autres problèmes se posent à l'éditeur d'aujourd'hui, qui sont cette fois engendrés par la multiplicité des copies. Pour en rendre compte, nous prendrons l'exemple du traité consacré par al-Sigzi aux démonstrations de certaines propositions des Eléments d'Euclide, Barahin kitab Uqlidis fi al usul[4]. Ce traité nous est parvenu par le biais de trois manuscrits :

- le n. 3652 de la Bibliothèque Chester Beatty, à Dublin, fol.17r 28v. Ce manuscrit, noté ici B, fait partie d'un recueil comprenant le premier volume d'un ensemble de deux volumes consacrés aux oeuvres géométriques d'al-Sigzi. Selon la table des matières, ce premier volume comprenait initialement 16 traités attribués au mathématicien persan. Les folios correspondants à 4 de ces traités et à la majeure partie d'un autre ont disparu, arrachés consécutivement au milieu du volume. La copie de l'ensemble consacré à al-Sigzi a été achevée, probablement à Bagdad, le vendredi matin 7 Ramaan 611 de l'Hégire, c'est à

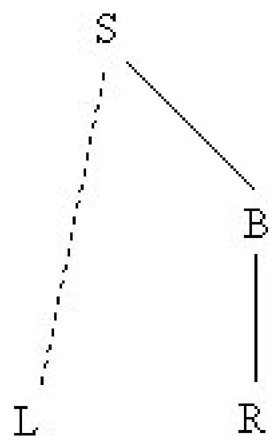
[4]Pour un aperçu sur ce texte, voir notre article : "Al-Sijzi et les Eléments d'Euclide : commentaires et autres démonstrations des propositions", Perspectves arabes et médiévales sur la tradition scientifique et philosophique grecque, A. Hasnawi, A. Elamrani-Jamal & M. Aouad eds, Peeters, Leuven-Paris, 1997, pp. 61 77.

dire le 9 janvier 1215. La copie du traité qui nous intéresse ici plus particulièrement a été réalisée, comme l'indique le colophon, à partir de l'original de l'auteur.

- Le n. 1191 de la collection Reshit, à Istanbul, fol. 84v-105v. Ce manuscrit, noté ici R, fait partie d'un recueil comprenant 14 des 16 traités mentionnés plus haut, dont les 4 traités et le fragment perdus du recueil précédent. Ce manuscrit appartenait à la collection du copiste Mustafa Sidqi et a donc été copié avant le milieu du XVIIIe siècle[5]. La date précise de la copie est incertaine, mais semble plutôt tardive. Les figures ne sont présentes que sur les folios 84v à 93v, les autres folios ne laissant que des espaces blancs prêts à les accueillir.

- Le n. 1270 de la Bibliothèque India Office, à Londres, fol. 87r-100r. Nous noterons ce manuscrit L. La copie est incomplète puisqu'elle s'interrompt brutalement aux neuf dixièmes du texte environ. Nous savons peu sur la tradition manuscrite dont elle est issue, et elle semble elle aussi tardive.

La comparaison méticuleuse du texte des manuscrits, en faisant abstraction des figures pour le moment, conduit au stemma suivant:



En effet, comparés à L, les manuscrits B et R présentent tous deux un certain nombre de lacunes, dont au moins trois de plus de 8 mots, ainsi que la répétition d'un paragraphe de 63 mots. D'autre part, comme le contenu des recueils semblait déjà le suggérer, R est bien une copie de B et de B seul. Aucune lacune, répétition ou erreur ne se trouve en effet sur D sans être également sur R, si ce n'est 5 corrections de type grammatical assez évidentes. Par contre, R montre vis-à-vis de B :

- 19 erreurs dans la désignation des objets géométriques ;

- 32 erreurs portant sur un mot ;

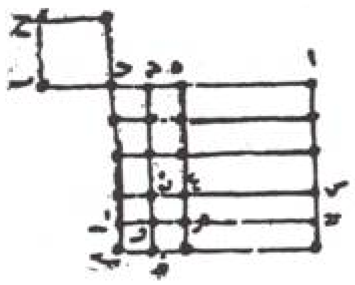
- 41 omissions (22 de 1 mot, 17 de 2 à 5 mots, une de 11 mots, une de 23 mots), sans compter les mots mis au dessus de la ligne dans D et qui sont, dans R, soit omis, soit mis sur la même ligne.

- 9 répétitions (5 de 2 mots, une de 3 mots, une de 4 mots, une de 7 mots, une de 8 mots).

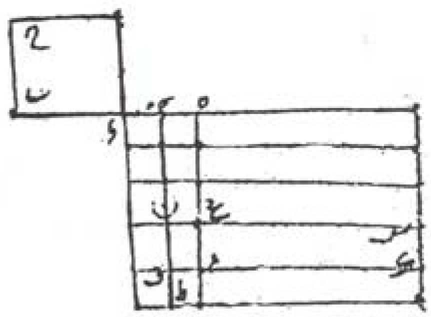
[5]Selon Roshdi Rashed, "La philosophie des mathématiques d'Ibn al Haytam", MIDEO, 20 (1991), pp. 31-231, en particulier p.33.

Or nous allons voir que cette filiation, dont nous venons d'étayer l'affirmation par le menu en considérant le texte seul, peut également se lire à l'aide des figures.

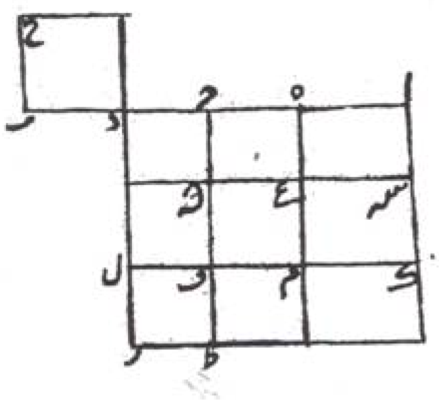
Remarquons avant toute chose que, d'une façon générale et compte tenu de la longueur du texte, le copiste de R est sans doute un bon copiste. Mais on sent aussi, eu égard aux erreurs commises, du peu de corrections apportées et de la nature de celles ci, qu'il n'est pas mathématicien, ou du moins, qu'il n'a pas porté sur sa copie le regard d'un mathématicien. D'une certaine manière, on pourrait considérer son travail comme un peu servile. Or ce trait est particulièrement sensible, précisément, pour ce qui touche aux figures. Il s'agit en premier lieu, d'une façon qui n'est pas sans rapport avec ce qui peut se produire pour le texte lui même, d'erreurs manifestes ou de tracés superflus qui sont reproduits sans que soit porté un regard critique. Ainsi de la figure accompagnant la seconde démonstration alternative proposée par al-Sigzi pour la proposition II-9 des Eléments[6]:



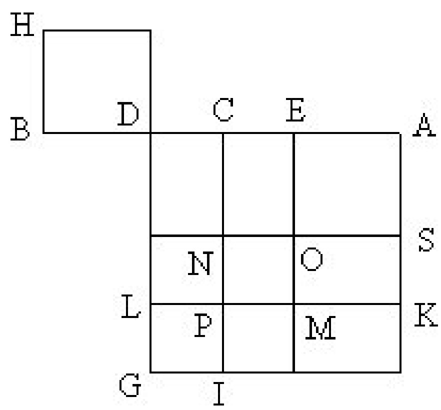
Manuscrit B



Manuscrit R



Manuscrit L



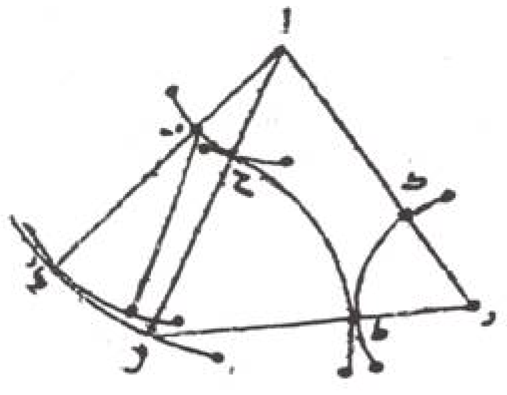
Notre reconstitution

On voit ici, sur les manuscrits B et R, deux lignes situées entre les droites AB et SN et parallèles à elles, mais qui ne se trouvent pas sur le manuscrit L. Or le texte, sur tous les manuscrits, tait le rôle de telles lignes, dont le tracé est sans rapport avec la démonstration. Cet ajout superflu permet ainsi, indiscutablement, de confirmer, au même titre que des particularités plus spécifiquement textuelles, le stemma avancé plus haut.

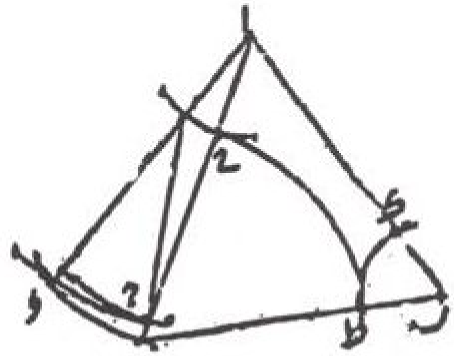
Mais ce trait du copiste de R peut être également perçu lorsque l'on considère l'allure générale des figures, d'une façon qui, sans doute, a peu d'équivalent avec ce que pourrait offrir la comparaison des textes proprement dits. Alors que ces figures peuvent avoir des allures

[6]B, fol. 19v; R, fol. 88v; L, fol. 90r .

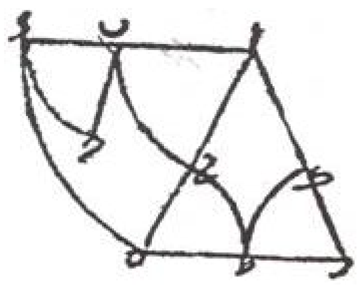
fort dissemblables lorsque l'on passe de B à L, elles sont au contraire généralement reproduites dans R avec le souci manifeste de ne pas s'écarter du modèle. Donnons-en un exemple, relatif à la quatrième démonstration donnée par al-Sigzi de la proposition I-2 des Eléments[7]:



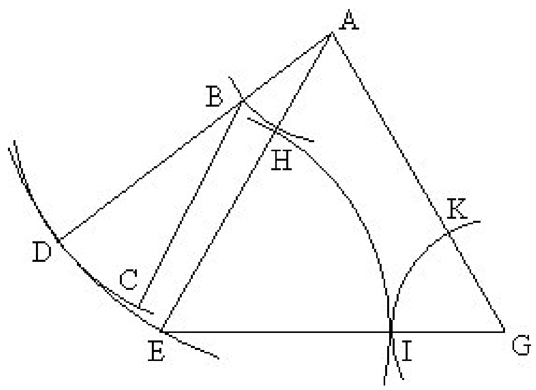
Manuscrit B



Manuscrit R



Manuscrit L



Notre reconstitution

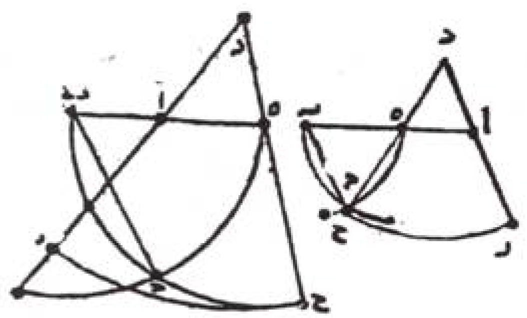
Notons en premier lieu que la dénomination du point E, absente sur B, est également absente sur R alors qu'elle ne l'est pas sur L, confirmant là encore le stemma avancé plus haut. Le point C, qui n'appartient au segment AE ni sur B ni sur L, lui appartient par contre sur R, introduisant une particularité qui relève d'un léger écart vis-à-vis du modèle. L'allure générale de la figure est toutefois bien la même sur B et sur R, alors qu'elle diffère très sensiblement dans le cas de L.

Il va sans dire que la figure de L " fonctionne " aussi bien que celle de B, et correspond au texte d'une manière identique. Dans une telle situation, le problème posé à l'éditeur est celui du choix qui doit s'opérer entre les diverses options, étant entendu que le contenu scientifique ne peut guider ici ce choix. Seule l'histoire de la transmission peut alors apporter des éléments de réponse.

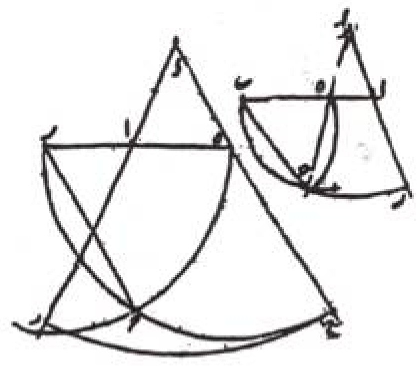
Dans cet exemple précis, nous aurions ainsi tendance à privilégier B - ce que nous avons fait dans notre reconstitution - parce qu'il a été copié directement à partir de l'original, alors que les nombreuses erreurs textuelles dont L fait preuve par ailleurs laissent penser que celui ci est le dernier maillon d'une chaîne plus longue. Mais cette préférence ne peut que conserver une certaine part d'arbitraire à laquelle il est difficile de se dérober.

[7] B, fol. 17v; R, fol. 85r; L, fol. 87v.

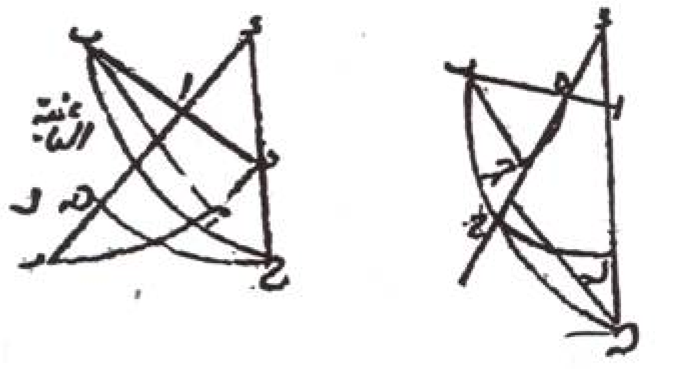
Un autre problème peut encore être illustré grâce à ce traité, celui de la généralité des figures, dont nous nous sommes entretenus plus haut. Prenons l'exemple de la troisième démonstration, par al-Sigzi, de la même proposition I 2, démonstration où apparaissent deux cas de figure[8] :



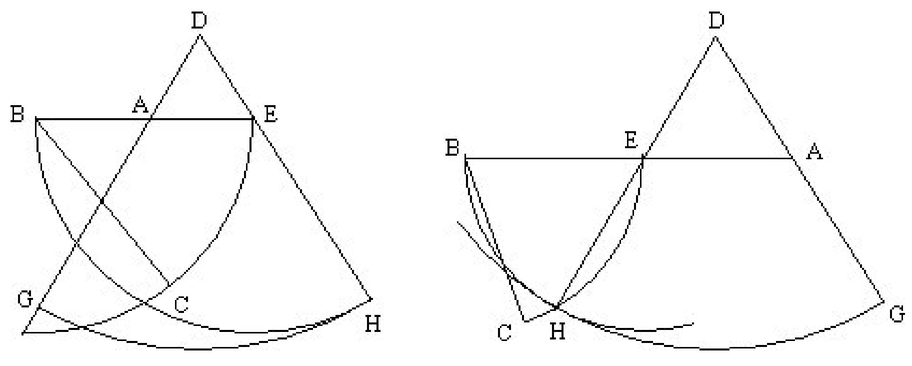
Manuscrit B



Manuscrit R



Manuscrit L



Notre reconstitution

Remarquons, sur les manuscrits B et R, que le point C, qui se trouve, dans les deux cas de figure, sur le cercle de centre B et de rayon BE, se trouve également, dans le second cas de figure (celui de gauche), sur le cercle de centre E et de rayon EB. Or cette particularité n'apparaît d'aucune façon sur le manuscrit L. En outre, elle repose sur une configuration des données initiales qui n'est en rien nécessaire : ces données, la droite AB et le point C, seraient alors telles que C se

[8] B, fol. 17r; R, fol. 85r; L, fol .87r.

trouverait sur la perpendiculaire à AB menée en A, ce que ne requiert aucunement l'énoncé. Dans les manuscrits B et R, qui diffèrent en cela du manuscrit L, le particulier représente donc le général dans le sens évoqué par nous plus haut. Quelle option l'éditeur doit-il prendre alors, sachant que, là encore, aucune représentation ne puisse être a priori rejetée ?

Pour notre part, nous n'avons pas repris cette particularité du manuscrit B dans notre reconstitution : en premier lieu parce que le manuscrit L semblait nous y autoriser; en second lieu parce qu'une telle option apparaissait mieux convenir aux normes en usage aujourd'hui. Ajoutons en outre que la figure du manuscrit B (comme celle de R) n'est pas exempte d'erreur, puisqu'elle intervertit à mauvais escient les places des points C et H dans le premier cas de figure, ce qui n'est pas le cas pour la figure du manuscrit L. Celui ci, pour être sans doute plus éloigné que B de l'original, ne devait donc pas être pour autant négligé. Mais on le voit là encore : les raisons qui animent les choix de l'éditeur peuvent être certes consistantes ; elles ne sont pas pour cela péremptoires.

Conclusion

Au terme de ce panorama, volontairement limité, rappelons le, au cas particulier de l'oeuvre géométrique d'al-Sigzi, il nous semble donc qu'une réflexion sur les figures, ou sur les normes à appliquer à leur propos de la part de ceux qui ont à en rendre compte, doive être poursuivie et approfondie. La tâche n'est sans doute pas aisée : d'une certaine façon, les figures constituent la part d'ombre des écrits géométriques. Elles y représentent en effet, de par leur nature même, ce qui est de l'ordre du non dit ; leur rapport à la pensée du mathématicien est à la fois profond et indicible; la transmission dont elles font l'objet engendre des variations qui sont parfois plus difficilement palpables que celles qui touchent au texte lui même. Mais cette difficulté n'est sans doute pas sans rapport avec ce qui, pour l'historien, constitue la valeur de l'enjeu : contribuer à pénétrer mieux encore au coeur de l'élaboration scientifique.

B.1.7 K. Chemla, *Editing the earliest extant mathematical figures from China*

The earliest extant figures of the mathematical corpus in Chinese were included by Zhao Shuang, in the third century, within the context of his commentary on the Han Canon of mathematical astronomy: *The Gnomon of the Zhou* (1st century B. C. E. or C. E.). And the earliest version of these figures that came down to us is to be found in the 1213 edition of the text by Bao Huanzhi. The first three of these figures (see attached reproductions, figures 1 to 3) open a development by Zhao Shuang entitled "Figures of the right-angled triangle, the square and the circle"[9].This piece of commentary is entirely devoted to the right-angled triangle that lay at the basis of astronomical measurements and, in it, Zhao Shuang lists abstract algorithms that allow yielding quantities attached to such a triangle, when knowing two such quantities. As will be shown below, several hints indicate that the first figure, the "Figure of the hypotenuse", must have resembled the original drawing used by Zhao Shuang. However, the two other figures were clearly damaged by the process of transmission. First, everything indicates that these two figures should be analogous. Their titles as well as their captions lead one to expect that they must have resembled each other. However, such is not the case of the received figures. Secondly, the captions mention gnomons, but not such shape appears on the figures of the 1213 edition.

How should we edit the damaged figures? Which criteria should we use in restoring them to their original shape? In the past decades, two editors have offered suggestions. Figure 4 shows

[9]One can also interpret the title as: "Figures of the base and the height, the square and the circle". See my translation of this piece of text in (Chemla & Guo 2004, 695-701).

the solution adopted by Qian Baocong in 1963, a solution that was accepted by Guo Shuchun and Liu Dun and included in their 1998 edition. Figure 5 shows the solution adopted by Li Jimin in 1990. In what follows, I shall indicate why I believe that Li Jimin's solution is more than probably much closer to the correct edition of the figures.

To do so - and this is the main point of my paper - I want to show that, in order to edit figures, one must inquire into the nature and use of these figures, in 3^{rd} century China. My claim is that it is only when we establish what these figures meant, and to which use they were put, that we are in a position to edit them in a correct way.

We shall discuss this issue on the basis of two documents: on the one hand, Zhao Shuang's commentary on *The Gnomon of the Zhou* and, on the other hand, Liu Hui's 3^{rd} century commentary on another Han Canon, devoted to mathematics: *The Nine Chapters on mathematical procedures*. O

Observing the received figures

Some preliminary observations on the figures of the 1213 edition will be useful for our purpose. First, note the fact that all figures are drawn on a support having a grid with unit squares. It is thus clear that the basic right-angled triangle for which the figures are drawn has the dimensions 3 (base), 4 (height) and 5 (hypotenuse). Secondly, note that all figures have the same frame: for each of them, the outer square has a side equal to the sum of the base (a) and the height (b), and a square whose side is the hypotenuse (c) is placed slantwise on it. Hence two of the three figures display shapes marked by grids oriented in two different ways. In the different figures, inside the square of the hypotenuse, some shapes are marked by thicker lines. Thirdly, captions comment on dimensions and colors. They indicate areas (the "square of the hypotenuse", the "vermillion area", the "yellow central area") and their values ("the square of the hypotenuse, 25, is vermillion and yellow"). They also indicate the three sides of the right-angled triangle and their values ("the base is 3"). Note that no point is named.

Why using Liu Hui's commentary on The Nine Chapters to discuss the figures?

Chapter 9 of *The Nine Chapters* is devoted to the right-angled triangle. The Nine Chapters having become, like *The Gnomon of the Zhou*, a Canon, commentaries on it were composed and some were selected by the written tradition to be handed down with the text of the Canon. The earliest of them all is Liu Hui's commentary, roughly contemporary with Zhao Shuang's commentary on *The Gnomon of the Zhou*. Chapter 9 consists of problems followed by algorithms solving them. In his commentary, Liu Hui systematically establishes the correctness of algorithms. In doing so, he concretely alludes to figures (tu), none of which was handed down to us. On the basis of Liu Hui's commentary, one is led to consider that the chapter devoted to the right-angled triangle in The Nine Chapters consists of two parts. The second one is characterized by the fact that it makes use of the "rule of three", even though in several distinct ways, whereas this dimension is completely absent from the first part.

What is crucial is that the first part, together with Liu Hui's commentary, happens to describe precisely the same algorithms as what can be found in Zhao Shuang's development entitled "Figures of the right-angled triangle, the square and the circle". Note that we have two texts that deliver the same algorithms, even though their shapes differ sharply. Zhao Shuang's text consists of abstract algorithms that follow each other and for which interpretations are occasionally provided explicitly with reference to the figures given at the beginning. However, *The Nine Chapters* describe algorithms within the context of problems that are for the most part particular. And, in chapter 9, these algorithms are regularly described with reference to the particular situation outlined by a problem.

In this part of the text, Liu Hui alludes to three figures.

One is clearly identical to the "Figure of the hypotenuse" as represented in the 1213 edition of Zhao Shuang's commentary: its outer square has a side equal to $a + b$. In it, the square of the hypotenuse is composed of a central yellow square having a side 3 equal to $b - a$ and of vermillion triangles. So not only is the inner structuring of the outer square the same, but even the colors are identical. This raises a question: why is it that the figure was drawn in exactly the same way? This already indicates that figures were probably not used as we do. Moreover, we thus understand that the edition of Liu Hui's figures depend on the edition of Zhao Shuang's and conversely. The two other figures to which Liu Hui alludes are symmetrical of each other and clearly bear some resemblance to the other figures originally contained in Zhao Shuang's text, if we rely on the captions. Liu Hui describes them as another way of placing the square of the base and the square of the height in the square of the hypotenuse. Again, the essential element is the square of the hypotenuse. The square of the base (respectively, of the height) is placed in one of its corners ("inside", says the text), with the color white, and the other area - equal to the square of the height (respectively, of the base)- , shaped as a blue-green gnomon, is placed around it, on the outside. As regards the pieces involved and the inner structuring of the square of the hypotenuse, this description fits with the captions of the received version of Zhao Shuang's "left" and "right" figures. In this case, however, the colors differ.

In fact, the received version of Zhao Shuang's figures is close to these: if we forget about the colors, it suffices to modify the place of the central squares, and, as a result, the figures correspond to both the captions and Liu Hui's description. These correspond to the figures as restored by Li Jimin.

If we consider the colors, these figures would be less stable than the first one, which is also correlated with the fact that they were damaged by the transmission. Both pairs of figures only use two colors (blue-green and yellow/white). In Liu Hui's description, the colors stress the similarities between the two figures. In Zhao Shuang's captions, the colors place emphasis on the equality of areas that have different shape. However, it is remarkable that these three figures and only them seem to be required, whether for the first part of chapter 9 or for the whole development by Zhao Shuang. This would entail that not only do the algorithms of Liu Hui and Zhao Shuang here coincide, but also their figures would do. But how are we to argue that such is the way in which Zhao Shuang's figures - and incidentally perhaps also Liu Hui's figures - should be edited? This leads us to the first argument in favor of Li Jimin's restoration of the figures.

Fundamental figures

Our first argument relates to the nature of these figures. In fact, if, as I believe, the three figures are the "Figure of the hypotenuse" and the set of two figures as described by Liu Hui, they are not only needed, but they also suffice, to establish the correctness of all the algorithms contained in either the first part of chapter 9 or the whole development by Zhao Shuang.

The reason for this is that each figure is the basis for proving the correctness of several algorithms. They are in this sense *fundamental figures*.

To understand this point, let us translate part of Zhao Shuang's development that relates to the "Figure of the hypotenuse": "Figures of the base and the height, the square and the circle.

1. Base and height being each multiplied by itself, summing up these (results) makes the square of the hypotenuse. Dividing this by extraction of the square root hence (gives) the hypotenuse.
2. Relying on the "Figure of the hypotenuse", one can further consider the multiplication of the base and the height by one another as 2 samples of the vermillion area (shi); doubling this (result) makes four samples of the vermillion area. One takes the multiplication by one another of the difference between the base and the height and itself as the central yellow area (shi). Adding one sample of the square (shi) of the difference (to the four obtained previously) also generates the square of the hypotenuse.
3. Subtracting the square of the difference from the square of the hypotenuse, halving the corresponding result, taking the difference as "joined divisor". [10] Dividing this by extraction of the square root yields, as a restoring, the base. Adding up the base to the difference hence (gives) the height.
4. The reason why, when doubling the square of the hypotenuse and subtracting[11] from it the square of the difference between the base and the height, there appears the square of the sum is that, if one examines it with the figure, doubling the square of the hypotenuse fills up the big outer square and there is a yellow area in excess. This yellow area in excess is the square of the difference between the base and the height. Subtracting from this (the former result) the square of the difference and extracting the root of the corresponding remainder hence yields the side of the big outer square. The side of the big square is the sum of the base and the height.
5. Carrying out the multiplication of the sum by itself and then subtracting it from the double of the square of the hypotenuse, extracting the root of the corresponding remainder yields the side of the central yellow square. The side of the central yellow square is the difference between the base and the height. Subtracting the difference from the sum and halving this (result) makes the base. Adding up the difference to the sum and halving this (result) makes the height.

All these algorithms, and in fact others, were proved to be correct on the basis of the "Figure of the hypotenuse". For this, the two pieces that the yellow square and the vermillion triangle

[10]This technical term refers to the coefficient in x of a quadratic equation. The coefficient in x^2 of such an equation was is, at that time, always implicitly taken to be equal to 1 and its constant term was called "dividend" (shi, same term as area, see my glossary in (Chemla & Guo, 977-978)). In this case, the introduction of the term "joined divisor" leads retrospectively one to understand that, in the algorithm (3) the area computed previously is the "area*dividend*constant term" of the equation, which states:

$$\frac{1}{2}(c^2 - (b-a)^2) = (b-a)x + x^2$$

The equation allows, when knowing the hypotenuse and the difference between the base (gou, a) and the height (gu, b), to determine the dimensions of a right-angled triangle. The base a is its solution, what Zhao Shuang describes as a "restoring" (fu). By reference to the "Figure of the hypotenuse", the area*constant term can be interpreted as a rectangle composed of two vermillion areas. The square of the unknown, a^2, leaves in it a rectangle, the dimensions of which are respectively x and $(b-a)$. This provides a geometrical figure of the quadratic equation that can be read on the figure . In ancient China, on the basis of this geometrical figure, quadratic equation was linked to square root extraction and solved as if extracting a square root, which explains that prescribing to solve the equation amounts to prescribing the extraction of a square root. For a more accurate treatment, see my introduction to chapter 9, in (Chemla & Guo 2004). Up to here, the passage can hence be interpreted with respect to the "Figure of the hypotenuse". There follows a passage relating to the other figures, which we skip, after what Zhao Shuang comes back again to "Figure of the hypotenuse".

[11]I adopt here an emendation suggested by Guo Shuchun and Liu Dun in their new critical edition of The Gnomon of the Zhou, in (Guo & Liu 1998, 3, 35 fn 23). All ancient sources have lie instead of the graphically similar character jian "subtract". They hence suppose that a copyist mistakenly copied one for the other.

constitute play a key part. Liu Hui uses the same figure in exactly the same way. This shows why the figure was so stable: it had to be the basis on which the correctness of a collection of algorithms could be proved and it was used to this end in a uniform way. This limited its possible variations.

This piece of evidence strengthens the thesis that the received version of the "Figure of the hypotenuse" must have looked like the original one. Conversely, it explains why the "figure of the hypotenuse" could be stable and why we can use the evidence provided by Liu Hui to bear witness to the 3^{rd} century shape of these figures.

In fact, all the algorithms contained in the pieces of text considered and the correctness of which could not be established with the "Figure of the hypotenuse" could be proved to be correct with the pair of other figures. So these figures too were fundamental in the same sense and this is why, most probably, variation on their shape was limited too. This supports Li Jimin's edition, as regards the structure of the figures.

There is an interesting difference between the "Figure of the hypotenuse" and this other pair: the proofs do not make use of the colors placed on the pieces. It seems that the function of the colors there was only to describe the inner structuring of the square of the hypotenuse. Can this explain the variation between the two commentators? Even though that could be the case, it is impossible to answer this question with certainty. In conclusion, it appears that the figures placed at the beginning of Zhao Shuang's development were to be the fundamental figures with respect to which the following algorithms could be exposed. In other terms, there appears to be a fundamental relation between these figures and the set of algorithms contained in either the first part of chapter 9, in The Nine chapters, or the whole development by Zhao Shuang. This has without doubt some relevance for understanding the mathematical texts we are dealing with. They do not consist in algorithms only, in relation to which figures would be ancillary: in these texts, algorithms and fundamental figures on the basis of which to account for their correctness form together a unified body of knowledge relating to the right-angled triangle. This is clearly demonstrated by the fact that the two texts by Liu Hui and Zhao Shuang contain both parts.

This feature may explain the position of the figures as the opening section of Zhao Shuang's text. These drawings seem to have been elaborated so as to constitute the 6 least amount of figures possible to lie at the basis of all the algorithms. This supports Li Jimin's edition as regards the number of figures. In contrast, Qian Baocong's multiplication of figures appears to me as revealing an anachronistic approach to figures, where there are as many figures as algorithms referring to them. The fundamental character of original figures is thereby lost At this point of my argument, we have accounted for the structure and number of figures in Li Jimin's solution. We still need to account for their basic shape.

The grid

Why is, in my view, Li Jimin right when, in contrast to Qian Baocong's, he restores figures on the basis of a grid with unit squares?

This argument requires widening the focus and looking at mathematical figures in China diachronically.

The earliest graphical evidence of figures in Chinese mathematical texts date from the 13^{th} century, whereas the earliest textual evidence comes from 3^{rd} century commentaries, which systematically refer to visual aids and their shapes. This may bear witness to a fundamental change in kind of mathematical texts between the 7th and the 13^{th} century. 13^{th} century mathematical texts essentially contain two types of figures: those illustrating a shape, and those on the basis of which to prove the correctness of algorithms. In contrast to the former, the latter are regularly represented with a unitsquared grid and they are used to interpret computations with areas. This is coherent with the marks 13^{th} century figures bear: either colors or characters are placed in the unit squares so as to allow evaluating the extension of a given area (see figure 6). This corresponds to the evidence provided by the text of the commentaries and hence conforms to the earliest textual evidence.

The same practice is attested to for solid geometry in the same way: for proving the correctness of algorithms, Liu Hui describes his use of blocks having unitary dimensions. These blocks are used to decompose the bodies examined or to analyze the volumes computed by the algorithms determining their volume.

All these elements seem to bear witness to a general use, from at least the 3^{rd} century onwards, of visual aids with unit components for proving the correctness of algorithms. They were used in geometry to interpret the meaning of the successive steps of the algorithms examined in the same way as problems were in other contexts. These remarks lead to conclude that, most probably, the earliest "figures (tu)" displayed the same kind of grid as the one attested to in the first graphical reproduction known. Li Jimin keeps this feature of the received figures, whereas, in my view, Qian Baocong reshapes them anachronistically.

Hypothesis for why the figures were damaged in transmission

We saw that there are fundamental reasons for the fact that Liu Hui's and Zhao Shuang's figures should be edited together. An important question remains: how are we to account for the fact that the transmitted figures examined in this paper were damaged? And, incidentally, are we to account for the fact that Liu Hui's figures got lost?

My hypothesis for this is to question whether the original tu's were drawn on paper for inclusion in the pages of a book or whether they could have been material objects.

Clearly the 13^{th} century graphical evidence for the drawings represent shapes cut in squared papers and placed on top of each other. Superposition plays an important role.

It is possible to account for the shape of the figures by making the hypothesis that originally there was a fundamental framework, on the basis of which several sets of shapes could be placed? The answer cannot be given with certainty, but several hints support this hypothesis. This then makes the problem of the edition even more difficult: are we to restore the 3^{rd} century figures as material objects?

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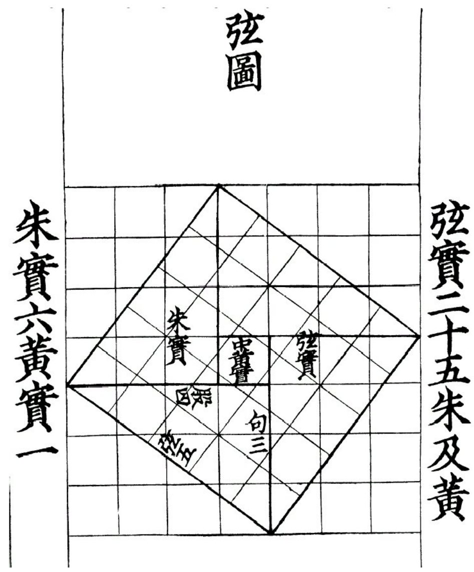


FIGURE 1: 1213 Edition of The Gnomon of the Zhou by Bao Huanzhi: The "Figures of the right-angled triangle, the square and the circle" opens with three figures

Translating the text on this diagram from top to bottom, right to left, the two characters at the top: xian tu, indicate that this is the "Figure of the hypotenuse". Now proceeding from right to left, we read: "The square (shi) of the hypotenuse, 25, is vermillion and yellow.// The square of the hypotenuse//The base is 3.//Central yellow area (shi).//(in horizontal characters) The height is 4.//Vermillion area (shi)//(slantwise) The hypotenuse is 5.//The vermillion areas are 6. The yellow area is 1.//"

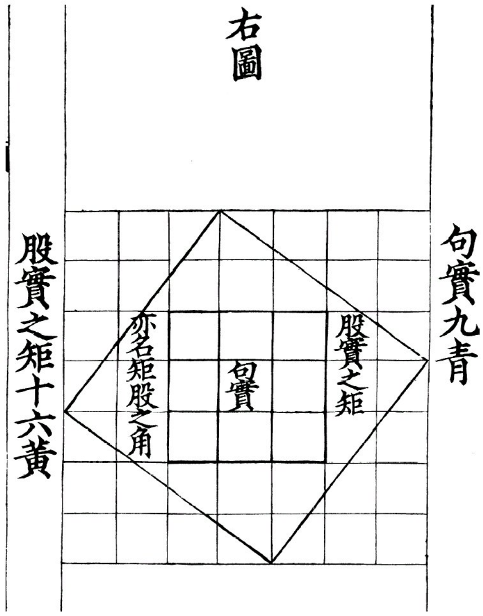


FIGURE 2: “Figure on the right”, 1213 edition

“The square of the base, 9, is blue-green.//The gnomon of the square of the height//The square of the base//Is also called the angle of the height as gnomon//The gnomon of the square of the height, 16, is yellow”

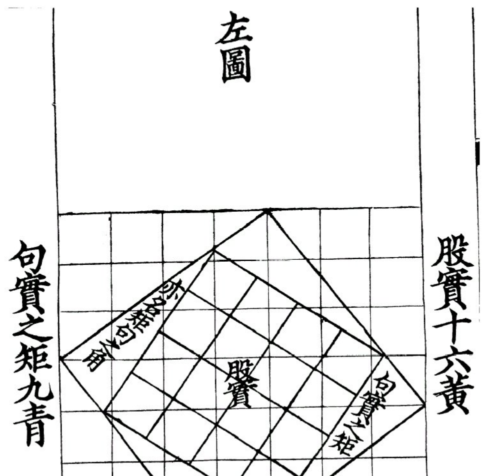


FIGURE 3: “Figure on the left”, 1213 edition

“The square of the height, 16, is yellow.//The gnomon of the square of the base//The square of the height //Is also called the angle of the base as gnomon//The gnomon of the square of the base, 9, is blue-green.”

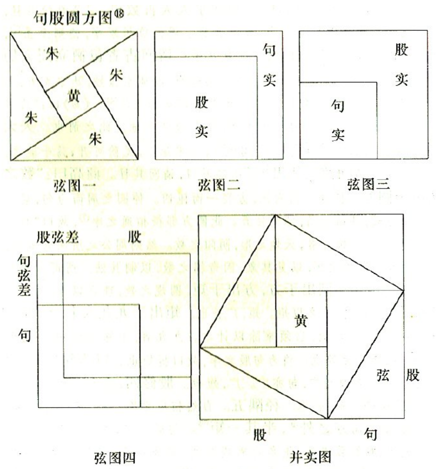


FIGURE 4: Edition of the set of figures by Qian Baocong 1963,
followed by Guo & Liu 1998

Captions

Figure of the hypotenuse 1: Bears names of colors: vermillion and yellow, in the center.

Figure of the hypotenuse 2: Opposes the "square of the height" to the "square of the base" in the shape of a gnomon.

Figure of the hypotenuse 3: Opposes the "square of the base" to the "square of the height" in the shape of a gnomon.

Figure of the hypotenuse 4: places the two previous figures on top of each other. This makes the difference between the hypotenuse and the height and the difference between the hypotenuse and the base appear.

Figure of the hypotenuse 5: Corresponds to the "Figure of the hypotenuse" of the 1213 version. It is called "Figure of the square of the sum" and only bears the color yellow for the middle square.

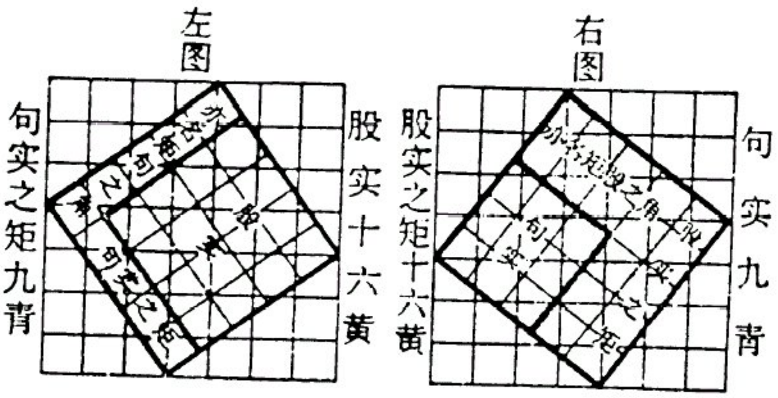


FIGURE 5: Edition of the figures on the right and on the left by
Li Jimin 1990

Caption for the "figure on the left", from right to left: "The square of the height, 16, is yellow.//(in the square, within the central square) The square of the height //(in the gnomon that is in the central square) The gnomon of the square of the base. Is also called the angle of the base as gnomon//The gnomon of the square of the base, 9, is blue-green."

Caption for the "figure on the right", from right to left: "The square of the base, 9, is blue-green.//(in the gnomon that is in the central square) The gnomon of the square of the height. Is also called the angle of the height as gnomon//(in the square, within the central square) The square of the base//The gnomon of the square of the height, 16, is yellow."

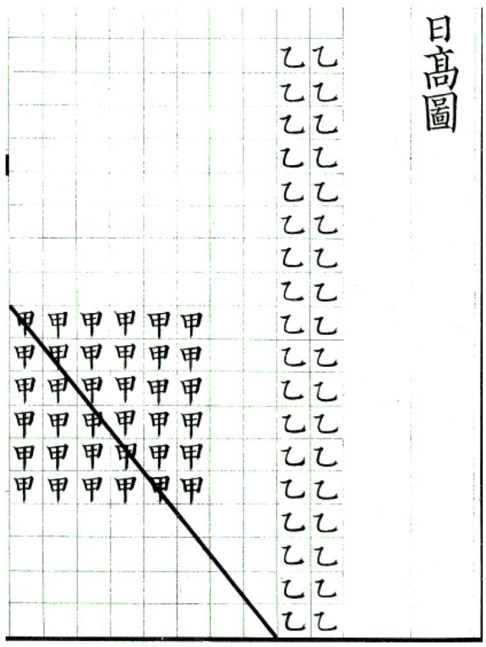


FIGURE 6: Figure of the height of the sun -1213 Edition. Left (top) and right (bottom) pages

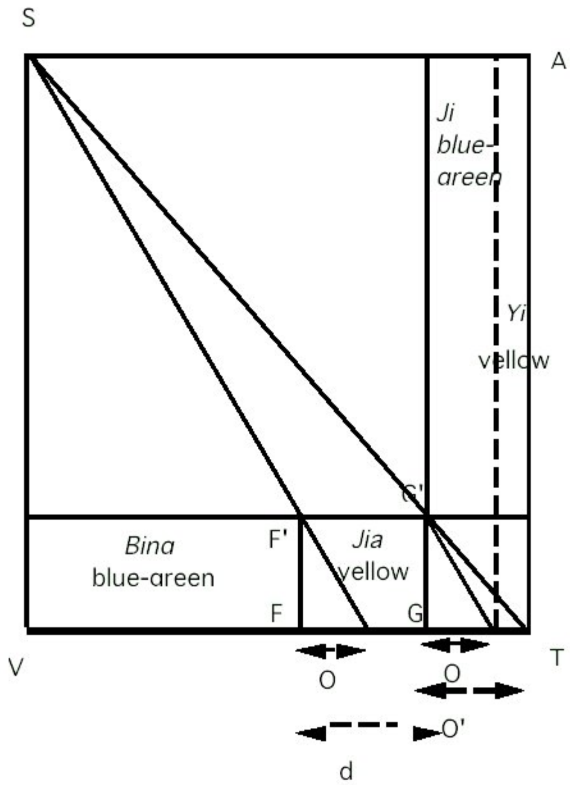


Figure 7: Idea of the figure of the height of the sun

O shadow of the southern gnomon at the summer solstice

O’ shadow of the northern gnomon at the summer solstice

d distance between the two gnomons

FF’ southern gnomon

GG’ northern gnomon

B.1.8 F. Ghione, *Diagrams in Menelaus’ Theorem*

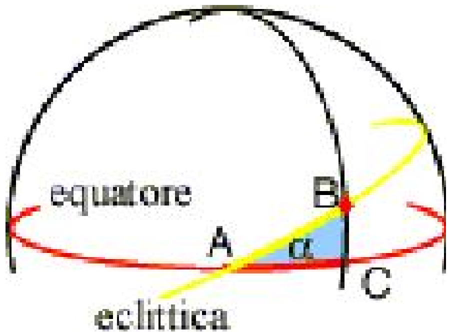
Certain mathematical statements can be interpreted in different ways, depending upon the context in which they appear. This condition could even alter the meaning and the value of statements: a theorem is interesting when it leads you to new connections or to that sudden insight which enables us to solve problems which were not clear. These connections create what we have called context, in which theorems are placed and can be explicated. In short, mathematical theorems have a vitality of their own and are able to change their characteristics according to the environment in which they are placed. It is true, of course, that certain statements hold more to this idea than others. Theorems are not constantly important in the same way, and we could say that they may follow the spirit of the the historical moment in which they are considered. The theorem of classification of regular polyhedra, for example, was extremely important, be it on mathematical or philosophical Ground, starting from Plato up to Pacioli’s divina proporzione. Today this theorem is very slightly considered, and when it is taught, its proof is avoided. We believe it very useful - for understanding mathematical statements and general mathematical thought - to insert theorems in the richest possible environment, considering them with reference to their history and the web of their possible meanings. In this sense we can be considered disciples of Federico Enriques, who uses the history of mathematics in the effort to show even advanced problems under various points of view[12] . A very illuminating example is given by the discussion of Bezout's theorem during his famous lessons on algebraic functions. During these lessons different - and for the most part incomplete - proofs and approaches to the underlying theory of elimination are illustrated; this theory was intuitively imagined, among others, by Stevin, Leibniz, Newton, Mac-Laurin, Eulero, Bezout, Jacobi and Cramer. Each approach, whether or not it may be decisive, sheds light on a different part of the theorem and may point to its possible application. In this way the theorem shows itself on a rich and varied background an thus allows deep and stimulating understanding. All of this is sometimes true even in much more simple circumstances. In some cases the figures which appear along with the mathematical material help insight and lead thought towards a specific meaning of the formal statements and are of great aid in synthetic understanding. Unfortunately, the use of figures in mathematics has been greatly reduced, due to extreme allegiance to rigorous structure and the tendency towards reducing mathematics to mere formal expression lacking in real meaning. Figures tend to be considered misleading and not worthy of high mathematical goals. This, in the final analysis, leads to the modern idea of publishing mathematical subjects with little regard for the design, the simple artistic beauty and the sense of figures. As in the example which will be discussed, such an attitude could also mean missing fundamental evidence of the original thought of the author. Although we do not intend in any way to deny the importance of rigorous mathematical structure, we firmly believe that the intelligent use of figures not only helps the understanding of mathematical statements, but leads our imagination to create situations and examples which the strict formal text alone cannot do. All that is dynamic and creative in thought is better represented by an image's continuous possibility of change in shape and position than a formal sequence of formulas. In this short discussion we would like to illustrate our thesis using an elementary but meaningful example: Menelaus' theorem starting from its nebulous origins up to Desagues. We will present three different formulations of this theorem, together with three figures which modify its gestalt framework. Each figure sheds light on particular boundaries which connect to the theorem itself. The whole of the different possible approaches results in rich, non - formal understanding, and greater concreteness.

Ptolemy and Menelaus

The first interpretation, Menelaus' and Ptolemy's original, is related to the problem of calculating the altitude of the sun on the horizon at midday in a given day of the year. A sphere represents the sky surrounding the earth placed at the center. The stars are projected onto the sphere and therefore are considered as points on the sphere itself. Three points may be considered as being "on the same line" when the three rays starting from the center of the sphere, in accordance with the geometry of vision[13] , are on the same plane. It follows that points on the same line are on the biggest circle and vice versa. It is possible to develop a spherical geometry, as Menelaus does, which reflects Euclid's plane geometry. In this model, the sun's movement in a year, can be represented by an "straight line"-the ecliptic- which makes an an angle with the equator that can be calculated. Menelaus' theorem makes it possible to calculate the arc CB of a right spherical triangle of which the arc AB and an angle α are known.

[12] F. Enriques, O. Chisini, Lezioni sulla teoria geometrica delle equazioni e delle funzioni algebriche, Zanichelli, 1918, vol I pag. 224, Vol II pag. 77

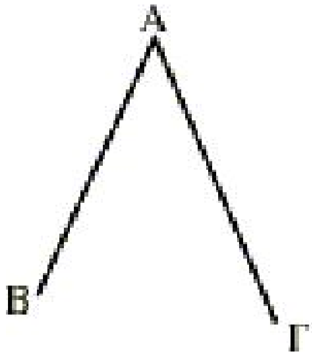
[13] L. Catastini, F. Ghione, Le geometrie della visione, Springer, 2004

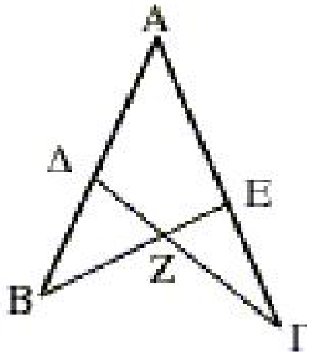


In the figure, if B is the position of the sun in a given day of the year, then we know the arc AB and the angle α and will be able to calculate geometrically the altitude BC of the sun for each day of the year. The theorem was of great importance in spherical geometry, for it made it possible to translate observable facts into quantitative ratios and therefore accurately and rationally predict things. The proof of this result is based on an analogous theorem of plane geometry -today called Menelaus's theorem- which does not appear in Euclid's *Elements*. The proof must have been well known. It may have been contained in Euclid's Porismi or in other lost elementary works. Menelaus himself seems to use it without proof: in fact it does not appear in the Hebrew codes which Halley[14] used in his edition of Menelaus's Spheres. The only direct source we have is Ptolemy's Almagest[15], in which we find a proof of this theorem (it appears as an introduction to the analogous theorem in spherical geometry) It is stated in the following way:

If two straight lines AΓ meet in A and are crossed by two other straight lines ΓΔ BE which intersect in point Z, I say that the ratio between the segment AE and the segment EΓ is equal to the ratio between segment ΔZ and segment ZΓ, composed of the ratio between segment BA and segment BΔ.

The image formed is that of two segments meeting in A to which are joined two other segments meeting in Z





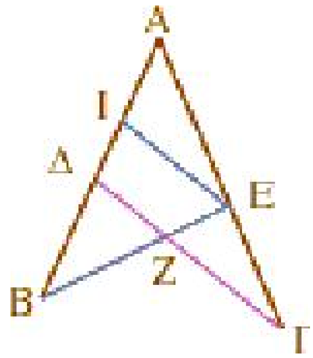
We can observe that it is explicitly stated that the two segments ΓΔ and BE are not parallel, otherwise the theorem would be exactly equivalent to Thales's theorem[16] . The Almagest theorem would then seem to be a generalization of Thales's, in an attempt to supply a tool in a spherical environment - where two straight lines always meet in a certain point Z - and thus substitute it to a certain extent. It seems meaningful that the above point is represented by the last letter of the alphabet, as if to suggest a special sense with regard to other points. The figures shown in the

[14]E. Halley, Menelai Sphaericorum, 1758, pag 82

[15]Ptolemy, Almagest, by G. J. Toomer, Duckworth, 1984, Libro I, n.13, pag. 64-69

[16]Euclide, Elemens, Book V, theorem 2.

various versions of the Almagest all have the same structure and the same letters. In the majority of cases the figures show segment EI as parallel to ΓΔ which is used to supply a proof of the formula of the ratios. In some manuscripts[17] , for example Gregorio Trapezuntio's XV century text, the two initial straight lines AΓ and AB are drawn with different colors so as to be distinguished from the others.



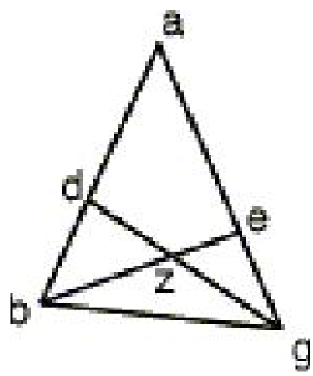
This theorem, and above all its analogous theorem in spherical geometry was highly considered by the Arabs, who variously translated and commented Menelaus' Spheres. Spherical trigonometry derives from this theorem, with all the important implications in astonomy. It is not by chance, then, that it also appears in Fibonacci, whose Arab sources are quite evident.

From the stars to the merchants

Fibonacci, in his Pratica geometrica refers to Menelaus' theorem using Ptolemy's letters, in their Latin vest, formulating it in a way that changes the gestalt framework[18] :

If in triangle abg. we draw the straight lines be. and gd. Coming from the angles b. and g. and if they meet in point z. then, if the ratio between ge. and ae. and between bd. and ad. are known, then we also know the ratio between bz. and ze. and between gz. and zd.

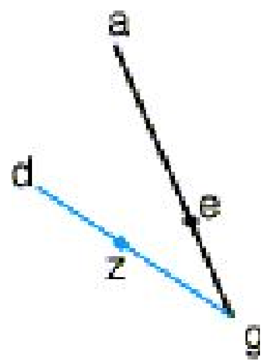
Menelaus' configuration, characterized by the intersection of four segments, becomes a fixed triangle which contains two segments, gd. And be. coming from the points g and b and intersecting in z:



[17]Gregorio Trapezuntio, Almagest, manoscritto del XV secolo, Biblioteca Apostolica vaticana, Vat. Lat. 2054, folio 16r

[18]si in trigo .abg. ab angulis .gb. egrediantur recte .be. et .gd. se inuicem secantes super punctum .z; et sit nota proportio ex .ge. ad .ae. et ex .bd. ad .da., erunt utique note proportiones .bz. ad .ze. et .gz. ad .zd. segue poi la formula del prodotto nei vari casi che possano considerarsi. In [11, Vol.II, pag.51]

We may note that the triangle is a curiously redundant figure with regard to the statement of the theorem, but its "closure" creates a good gestalt structure, easily remembered and which guides the reconstruction of possible numerical relations. The first relation supplied by Fibonacci is is the one that expresses the ratio ga:ea as a compound ratio of gd:zd with bz:be. The text probably indicates a way to memorize the formula. We start by dividing segment ga with a point e according to the ratio ga:ea and then we take the segment riflesso in toto gd and the entire reflected segment is considered with regard to its ratio with regard to its part zd.



We then draw the straight lines ad and ez which meet at the third vertex b of the triangle, thus creating the segments bz and be which create the ratio. Compound ratios were of great practical importance, being that in this way it is possible, for example, to calculate the value of one type of currency in relation to another type once you know its value with regard to an intermediate type and the value of the intermediate with regard to the one you are looking for. These relations were used mechanically to solve commercial and numerical problems; they are reported in Liber Abaci and are called regula baracti or regula sex quantitatum, and this because given five of six quantities you can find the value of the sixth. This approach to the text of the theorem was more useful to merchants for memorizing purposes than to mathematicians. Here Menelaus' theorem is like a geometrical transcription of the product of two ratios which results in very strong geometrical demonstrations with much practical value.

In Desargues's garden

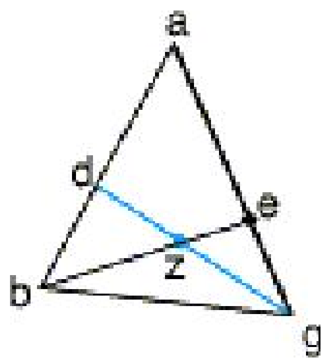
Desargues work is still today obscure in many aspects. This is true because he was not highly considered in his time and there is a great lack of studies which could have shed light on his efforts. Particularly, his most important work, Brouillon project d'une atteinte aux evenemens des rencontres d'un cone avec un plan, was published in 50 copies in 1639 and were almost immediately lost[19]. Up until a short time ago, the only trace we had of his work was a copy handwritten forty years later by La Hire, which M. Chasles found by chance in 1845. It can be found today in the library of L'institut de France in Paris. N. Poudra[20], in 1864, published a first edition of the works of Desargues which had up till then been found. In 1922 an original copy of the Brouillon was found, although without figures, and was used by R. Taton[21] to publish a new edition of Desargues' work. In comparing La Hire's copy with the original text one finds many differences. Many notes and corrections by Desargues himself are missing in La Hire's

[19] Vedi anche L. Catastini, Il giardino di Desargues, BUMI, serie VIII, Vol. VII-A, 2004, pag. 321-346 g e L. Catastini, F. Ghione Nella mente di Desargues, In corso di Stampa sul BUMI, sez. A

[20] N. Poudra, Oeuvres de Desargues réunies et analysées, 2 Volumi , Paris, 1864

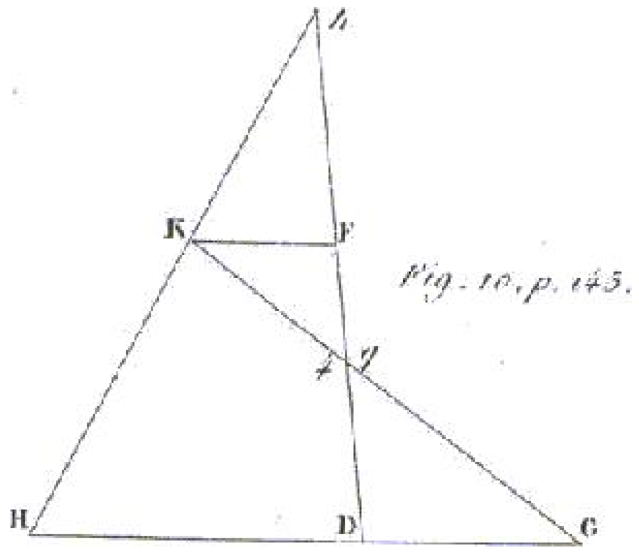
[21] R. Taton, L'oeuvre mathématique de G. Desargues, Presses Universitaires de France, 1951 (seconda edizione rivista 1981).

copy. The numbering of the figures is not the same: in the original text figure one corresponds to figure 10 in La Hire's copy, as if the fist nine were of little importance and just casual additions to the text. In our opinion the first important figure is the one related to Menelaus' theorem, which, in the projective environment which Desargues imagined, acquires new meaning. The new terms used by Desargues are considered by some to be "barbarian" (trunk, knots, branches, twigs), but we believe that they are useful in creating a new environment which aids the understanding of projected space. In this sense figures become particularly important. The reconstruction we find in both Poudra's and Taton's editions is not very faithful to the original, and in any case does not add anything new to the text. The hypothesis we suggest makes Desargues' strange language clearer and opens the way towards a dynamic view which is surely very important for the understanding of the idea of involution, a concept which is the basis of the new projective approach. This case shows how the little interest mathematicians and historians have for figures in the end weakens the scientific content of the text. We will start by considering Desargues' statement of Menelaus' theorem as we find it in Poudra (vol 1, page 145) together with the figure.



> Quand en une droite H, D, G, comme tronc, a trois points H, D, G, comme noeus, passent trois droictes comme rameaux dÈployer HKh,Dqh,G4K , le quelconque brin Dh, du quelconque de ces rameaux D4h contenu entre son noeu D, & le quelconque des deux autres rameaux HKh, est a son accuplÈ le brin Dq contenu entre le mesme noeu D, & l'autre troisieme des mesmes rameaux G4K en raison mesme que la composÈe des raisons d'entre les deux brins de chacun des autres deux rameaux convenablement ordonnez, à scavoir de la raison du brin comme Hh, au brin comme HK, & de la raison du brin comme GK, au brin comme G4.

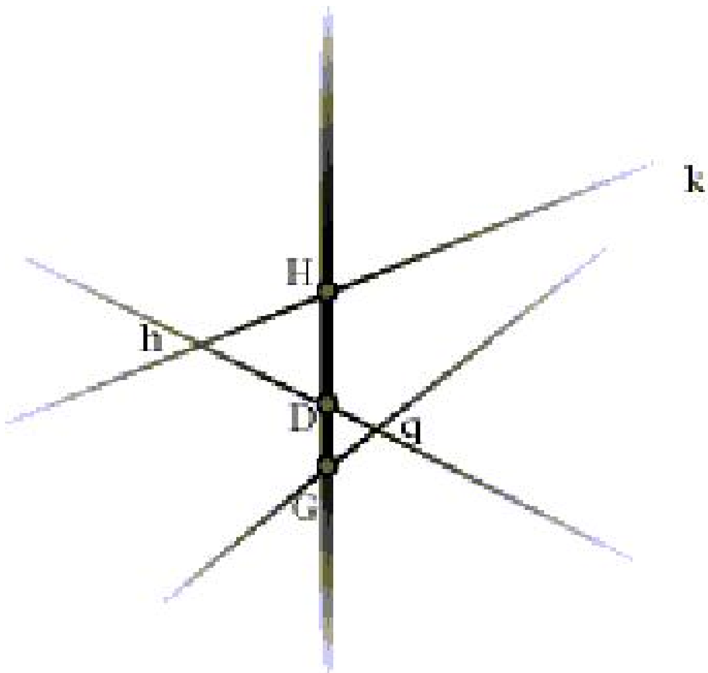
As can be seen one same point is represented, surely by mistake, with two different symbols: first with q then with number 4. We strangely also see the same mistake, however, in the figure in which we find two signs near the intersection point of the two branches GK and Dh. The letter q then disappears and the point is represented by the number 4. In Taton's version - written according to the original published copy, but without drawings - the error is "corrected" in the figure also and he decides to use only the number 4, which appears more frequently, and writes in sometimes different but unimportant usage of language ("droicte" instead of "droite", "poinct" instead of "point", etc.).



The least that can be said is that it lacks elegance. Being that all the points (they are only six) are identified with latin letters, why should one be represented by a number? Given that the handwriting makes q and 4 look much alike, we think that the most reasonable way of indicating that pointis to use q. It is also true that the two fonts could easily be mistaken in La Hire's manuscript and Poudra, in his edition, which was the one Taton took as his basis, probably just took an unreasonable decision, not paying much attention to this formal aspect, apparently of little importance. This oversight leads us to the consider that there may be other errors in the interpretation of the missing figure, which could be much more important than a simple diagram, if considered in a different way. We don't know much about Desagues, but it can surely be said that choosing to use that very strange language makes of him a very particular type of mathematician. The only figure we have - attributed to him - illustrates his method for perspective representation, on a square plane, of any point in the space coordinates (x,y,z). The figure is indeed very beautiful and meaningful, and we cannot exclude that the few figures which were part of the Brouillon were more significant than a quich drawing done by a copier and then printed by Poudra end taken up by Taton. In any case, the figure as it is here represented does not refer in any way to the new concepts which were presented by Desargues (trunk, branch, knot, twig); but these terms are used in the text. The figure is only a banal outline, following the tradition of the times. We ask why Desargues would have used a different language to express something which was well known (in Ptolemy, for example, who is referred to by Desargues) and in may others. We hold that Desargues is trying to say something new about Menelaus' theorem and he needs new concepts, words and figures. Let us suppose that the letter K, which is capital in the two printed texts, were small. Even the difference between k and K, just like the difference between 4 and q is not graphically important. There is no formal question here, but semantically speaking, it makes a very big difference. With the changes we refer to above, Desargues mathematical statement would be the following:

> Quand en une droite H, D, G, comme tronc, a trois points H, D, G, comme noeus, passent trois droictes comme rameaux dÈployer Hkh,Dqh, Gqk , le quelconque brin Dh, du quelconque de ces rameaux Dqh contenu entre son noeu D, & le quelconque des deux autres rameaux Hkh, est a son acculÈ le brin Dq contenu entre le mesme noeu D, & l'autre troisieme des mesmes rameaux Gqk en raison mesme que la composÈe des raisons d'entre les deux brins de chacun des autres deux rameaux convenablement ordonnez, à scavoir de la raison du brin comme Hh, au brin comme Hk, & de la raison du brin comme Gk, au brin comme Gq.

We accompany this statement with the following figure:



$$\frac{Dh}{Dq} = \frac{Hh}{Hk} \times \frac{Gk}{Gq}$$

This image, with the straight lines fading away and giving the idea of their extending towards infinity, is not meant to be a miraculous appearance od Desargues' lost image, but it is meant to be an emphatic example of the meaning, in our view, that the so-called barbarian terms now could have. The knots that live on the trunk (which is verical, as is the case with every type of plant) are represented with capital letters. These knots give rise to branches which meet in "points", which could be "buts", as Desargues calls them, or, as we would say today, points to infinity. All in all, objects possibly different, not like usual points, new objects which are represented by small letters. Please note that Desargues immediately states the famous potulation: two straight lines on a plane have always one common point. The strange concept of "twigs" also becomes meaningful: only the twigs, that is the segments that are born in the knots, are part of the formula of ratios. Segments qk, hq and hk don't have any particular name, they are not important objects to memorize and define with a new name. In the same way, the triangle hqk is not mentioned and not made evident in any way. The triangle is not a Desargues object, it is not a projective object: the vertex k could be projected to infinity and the triangle could disappear, but the theorem woud still hold. In all of the Broullion the term "triangle" appears only once as an example - in a clearly metrical situation - of certain practical applications of the new theory. Moreover, the idea that the letter k must be small and not capital is supported by a strange note, a sort of "errata-corrige which Desargues adds to the text of the Menelaus statement before printing. Desargues feels the need to specify that "generally K's are capital letters" (Taton page 126). Since, from that point onwards K's are alway capital letters, the meaning of this note is not very clear, if not for the fact that in this case (Menelao) only the letter k is used in its proper vest, that is as a small letter.

B.1.9 P. Radelet-de-Grave, *L'édition des figures manuscrites des Bernoulli*

Les plus importants manuscrits, laissés par les Bernoulli, cette dynastie de scientifiques bâlois vivant entre 1654 et 1807, sont au nombre de trois. Ils traitent de mathématique et de physique. Ils offrent des problèmes éditoriaux très différents.

Les *Meditationes* de Jacob écrit de 1677 à 1705

Il s'agit d'un autographe de Jacob (1654-1705), les *Meditationes, Annotationes, Animadversiones Theologicae et Philosophicae, a me JB. concinnatae et collectae ab anno 1677*[22]. Ce journal scientifique où il notait certains calculs comme les brouillons de certains articles l'a suivi toute sa vie de 1677 à sa mort. C'est un autographe, unique, dont certains morceaux choisis par lui ont été publiés dans les *Opera* de Jacob, publiés à Genève en 1744 sous le titre choisi par Jacob lui-même de *Varia Posthuma* mais bien après sa mort. Cette publication n'a donc pas été contrôlée par l'auteur. Par contre d'autres *Meditationes* ont été publiées dans des journaux du vivant de l'auteur, donc sous son contrôle. Le statut des figures reproduites là est donc très différent. Nous sommes donc en possession d'une seule version manuscrite, autographe, des figures de ce manuscrit et de quelques figures publiées du vivant de l'auteur. Finalement, il y a quelques figures qui ont été publiées bien après la mort de Jacob. Il faut donc, a priori, il me semble privilégier la publication des figures autographes originales. Malheureusement certaines d'entre elles sont de très mauvaise qualité. Or nous désirons à la fois respecter l'authenticité et faciliter la compréhension du texte par le lecteur. Dans le cas des articles publiés du vivant de l'auteur, il a été décidé de reproduire à la fois la *Meditationes* et le texte publié. Le problème est alors puisque le lecteur trouve à la fois la figure originale et la figure publiée. Une note peut si nécessaire assurer le renvoi de l'une à l'autre. Dans les autres cas deux solutions se présentent. De toute façon reproduire la figure autographe originale puis la compléter en note soit par la reproduction de la figure publiée dans les Opera, soit par un dessin moderne. Ce dernier point est un choix qui peut faire l'objet d'une discussion.

Le Manuscrit de Saint Pétersbourg de Daniel écrit aux environs de 1731

Le manuscrit contient la version préparatoire de l'*Hydrodynamica* de la main de Daniel Bernoulli (Portait) (Page de titre du manuscrit de Saint Pétersbourg). Cette version est différente de la version publiée à Strasbourg en 1738, du vivant de l'auteur. Comme cette différence concerne les principes de la dynamique et porte donc à conséquence, il a été décidé de publier intégralement le texte publié de l'*Hydrodynamica* (ce qui est déjà fait dans le volume 5 des *Werke* von Daniel Bernoulli) et la version manuscrite laissée par Daniel à Saint Pétersbourg. Les lecteurs auront donc les deux versions, mais elles seront dans des volumes différents. Quels liens faire et comment les établir avec les figures de la version finale publiée quelques années plus tard ?

Les *Leçons* à de l'Hôpital de Johann écrit en 1691-92 et annoté jusqu'à sa publication en 1742

C'est sur ce manuscrit que nous nous concentrerons. Il s'agit du texte des leçons données par Johann au Marquis de l'Hôpital à Paris entre 1691 et 1692. Nous sommes en possession d'un seul manuscrit du calcul différentiel mais de 6 manuscrits plus une publication du calcul intégral. Ces manuscrits ne sont pas tous complets. Aucun n'est autographe, l'un d'entre eux est annoté par Johann et semble avoir servi de base à la publication dans les Opera. De plus un manuscrit de la nationale à Paris contient les figures seules. Comment sélectionner les figures à reproduire parmi ces sept versions de figures ? Ces figures peuvent-elles nous aider à dater ces différents manuscrits ?

Les leçons sur le calcul différentiel Nous ne possédons qu'un manuscrit des leçons sur le calcul différentiel : LIa 6 : Johannes I Bernoulli, Lectiones de calculo differentialium et integralium, dit

[22] Manuscrit, Universitätsbibliothek Basel, L Ia 3, publié partiellement dans Jakob Bernoulli, *Werke* 1-5.

manuscrit A. Ce manuscrit est de la main de Nicolaus I, il contient le Calcul différentiel et les 11 premières Leçons du calcul intégral. La partie concernant le calcul différentiel a été reproduite par Schafheitlin en 1922. Selon Spiess, suivit Hess, la copie date de 1705 mais je crois que l'étude de la partie de ce manuscrit qui concerne le calcul intégral est susceptible de modifier cette date. Les figures sont intégrées dans le texte. Ce sont les seules figures que nous ayons pour cette partie des Leçons.

Les Leçons sur le calcul intégral Nous sommes en possession de 7 manuscrits,

1. quelques pages de LIa 6 déjà mentionné, de la main de Nicolaus I . Les figures y sont intégrées, mais le manuscrit est largement incomplet. Il s'arrête à la 11eme leçon sur le calcul intégral qui en compte 59.

2. LIa 7 : Johannes I Bernoulli, Lectiones de calculo differentialium et integralium, dit manuscrit B. De methodo integralium, copie d'une main qui n'est ni celle de Johann ni de Nicolaus I. Il contient 46 feuillets plus une planche n. 47 de figures c'est-à-dire Leçons 1 à 26, 33 à 44 et 46. Les figures sont intégrées plus une planche.

3. LIa 8 : Johannes I Bernoulli, De methodo integralium, dit manuscrit C. Copie d'une main inconnue (Staehelin) avec des parties et des corrections de la main de Johann. Cette copie a probablement servi à la publication des Opera. Ce manuscrit est le plus ancien. Les autres tiennent compte de certaines corrections qui ont été faites dans la marge de ce manuscrit de la main de Johann Bernoulli, de Stähelin et d'une autre main inconnue. C'est aussi dans la marge que se trouvent les figures qui sont de la main de Stähelin (cf lettre de Bernoulli à Montmort 29 septembre 1718).

4. LIa 9 : Johannes I Bernoulli, Praelectiones de calculo integrali quas in usum Ill. Marchionis Hospitalii Parisiis conscripsit Vir Celeberrimus Joh. Bernoullius, dit manuscrit D. Contient l'ensemble des leçons publiées mais est postérieur à la rédaction. Il n'y a aucune figure.

5. LIa 9a : Johannes I Bernoulli, De methodo integralium, Has lectiones scripserat Cl. Joh. Bernoullius in usum Ill. Marchionis Hospitalii cum Parisiis esset 1691-1692. Copie postérieure à la rédaction. Il n'y a aucune figure.

6. B.N. Fds Latin 17860, fol. 91-240 ; 251-252. Contient les leçons 1 à 35 et 46-59, les pages 557-558 des Opera n'y sont pas reprises. Costabel pense que les Leçons 36 à 45 sur la caténaire ont été perdues par Montmort parce qu'il manque 10 folios. Ces textes ne figurent pas dans l'analyse des infiniment petits de l'Hopital non plus. Ce n'est pas possible car les 10 folios perdus se trouvent au milieu du chapitre consacré à la méthode inverse des tangentes et que la matière correspondant à ces 10 folios manque également. Par ailleurs, des copies des articles de Johann sur la caténaire et la voilière font suite dans le manuscrit Fds latin 17860 au texte des Leçons. Ces articles étaient déjà publiés lorsque Johann part à Paris. Malheureusement, il contiennent les énoncés des propriétés mais pas les démonstrations. Costabel prétend qu'il y avait une copie à Bâle des textes sur la caténaire qui a circulé parmi les Géomètres. Ce pourrait être le manuscrit Lia 7. Dans le manuscrit Fds latin 17860 les figures sont rassemblées vers la fin du manuscrit, avant des errata de la main de Byzance.

7. B.N. Fds Fr. 24235 Ce manuscrit contient aux Fol 14-27 les figures manuscrites de la main du Père Byzance des Leçons. Sans aucun texte. Nous sommes donc en possession pour les Leçons sur le calcul intégral de 3 jeux complets de figures manuscrites plus deux

jeux réellement incomplets dont un de la main de Nicolaus I, secrétaire de Johann. A cela s'ajoutent les figures publiées en 1742 dans les Opera, encore du vivant de Johann.

Ce que nous savons des copies de ce texte par des sources historiques : Johann explique qu'il faisait copier ses notes avant le cours pour ensuite donner sa copie à de l'Hôpital.

> J'avais cependant la prévoyance de les faire copier par un ami qui logeait avec moi, avant de porter les originaux à Mr. le l'Hospital[23].

Dans l'idée de Spiess, cette copie parisienne a été ramenée à Bâle par Johann et c'est sur celle-ci qu'il a mis des notes et que c'est elle qui a servi de base à l'édition des Opera en 1742. Je suis d'accord avec lui Johann a donc corrigé cette copie à de nombreuses reprises.

> Pendant notre sejour à Ouques, je n'ai pas manqué de donner à Mr. de l'Hospital de nouveaux mémoires ecrits toujours de ma main, à mesure que j'en trouvois de la matiere et qu'il m'en fournissoit lui meme l'occasion par toutes sortes de questions. Faute de copiste, je n'ai pas pris de copie de ces mémoires ecrits à Ouques; mais pour les leçons faites à Paris, un de mes Amis de Bale qui logeoit chés moi, avoit la complaisance de me copier chacune avant que je la portasse chés Mr. le Marquis, ainsi je les ai conservées toutes[24].

Le 28 octobre 1718, Montmort écrit à Johann.

> C'est d'un ami [Stähelin] qui étoit à Paris avec vous, & qui copioit vos leçons pour Mr. de l'Hospital que le Père Reyneau a tiré son manuscrit, dont j'ai bien remarqué quelques petits lambeaux dans son livre Analyse démontrée : Le Père Bizance en avoit aussi un. Comme je pressois Mr. le M. de l'Hospital de me le prêter; il me donna une lettre pour le Père Bizance, par laquelle il le prioit de me prêter le sien; mais apparemment le mot était donné pour n'en rien faire, car je ne l'eus point; le P. Reyneau me prêta le sien environ un an après[25].

Comment j'imagine que les choses se sont passées.

Johann prépare ses cours chez lui, fait ensuite copier le texte par Stähelin ou le recopie lui même. C'est le manuscrit C ou Lia 8. Le jour du cours, il donne la première copie à l'Hopital et il relit le texte en présence de l'Hôpital. A cette occasion, il constate certaines erreurs qu'il corrige le soir même dans la grande marge laissée par Stähelin et probablement sur le manuscrit de l'Hopital également. Je ne crois pas que soyons en possession de ce dernier. Lorsqu'il part à Oucques, il laisse le manuscrit à Stähelin (hypothèse de Spiess) qui le prête à Byzance qui le fait copier par Carré. Carré intègre donc ces premières corrections dans sa copie. C'est le manuscrit BN Fds latin 17860. Pendant que Reyneau court à Oucques pour mendier une copie (Opera t. II, p. 76) qu'il ne recevra pas et prendra sans doute sur celle de Byzance. A moins que la copie que Reyneau trouvera enfin (cf Cosatbel) ne soit celle de Carré Byzance). Pour sa part Johann retravaille certaines choses avec l'Hôpital à Oucques, sur base de la copie originale de ce dernier,

[23] Johann Bernoulli, *Autobiographie*, tome II, p. 75.

[24] Johann Bernoulli à Montmort, lettre du 21 mai 1718, Manuscrit de la bibliothèque universitaire de Bâle.

[25] Cet extrait de lettre a été repris dans Op. CXX, Johannis Burcardi, Basileensis, Epistola ad Virum Clarissimum Brook Taylor, J.V.D.R.S.B. Soc., Johann Bernoulli, *Opera Omnia*, Tome II, p. 509.

et note quelques autres erreurs qui parviendront à Byzance après la copie du manuscrit. D'où les folios séparé de Byzance avec le texte exact des deuxièmes notes reportées par Johann dans la marge du manuscrit C ; Lia 8.

Les feuillets manquants du manuscrit de Byzance qui devraient être perdu dans le manuscrit Reyneau. La thèse de Costabel (p. 157) qui estime que l'existence de deux manuscrits (Byzance et Reyneau) n'est pas attestée. Des papiers de Byzance devenu fou auraient pu être emportés par Reyneau.

De retour à Bâle Johann va toujours conserver le manuscrit C et y noter encore quelques corrections. Ex. Une note marginale qui cite la manoeuvre des vaisseaux qui date de 1714. On devrait placer les copies bâloise chronologiquement en fonction de la prise en copte des différentes corrections. Et de plus classer ces corrections aussi. En tout cas il reste des dernières corrections qui apparaissent dans les Opera et qui ne se trouvent dans aucun manuscrit y compris C. Johann a donc encore vu une dernière copie de ce manuscrit C qui a disparu. Il était impossible d'envoyer ce manuscrit tel quel à l'imprimeur.

Johann a maintenu l'ordre initial des *Leçons* en ne rapprochant pas les additamentum sur les caustiques des autres chapitres sur ce sujet contrairement au manuscrit parisien. C'est selon moi pour des raisons de priorité. Il faut dire que ses corrections ne sont pas des remaniements mais des petites corrections de fautes manifestes.

Figures

Nous sommes en possession de 6 jeux de figures (il m'en manque un. La BN de Paris ne permettant la photographie que par son service photographique et par volume entier, et moi refusant de payer la photographie de 170 feuillets pour 13 qui m'intéressent).

Les lettres des figures de tous les manuscrits sont identiques (à de très petites erreurs prêt).

Les figures du manuscrit C, Lia 8 ne sont pas de Johann.

Bernoulli à Montmort 29 septembre 1718.

> Je consens volontiers que quelqu'un prenne la peine de les faire imprimer, je lui prêterai ma copie qui est la plus ancienne puisqu'elle a été faite immédiatement sur les originaux à mesure que je les avais composés, mais ayant été copiée par une personne qui n'entendait nullement la matière, elle est remplie d'une infinité de fautes, tant dans les figures (qui sont très mal faites) que dans les calculs. Il faudrait donc les corriger soigneusement et puis faire un extrait de mes lettres à Mr. De l'Hôpital.

J'ai collé toutes ces figures en parallèle dans un dossier papier que j'apporterai. Il eut fallu les scanner mais je n'en ai pas la patience. Je ferai une sélection d'images me semblant plus intéressantes.

B.1.10 S. Probst, *Editing Mathematical Drawings from Leibniz's Manuscripts*

The mathematical papers of Leibniz

The figures, diagrams and drawings in the mathematical manuscripts of Gottfried Wilhelm Leibniz offer a variety of mathematical, technical, and philological problems for a critical edition. One of the main reasons is the special, quite unique character of Leibniz's mathematical papers: With the exception of a relatively small part the bulk of the extant manuscripts do not consist of elaborated writings, destined for publication, but of Leibniz's original work sheets, notes and attempts of calculation and construction. If we confine our consideration to the period of Leibniz's sojourn at Paris from 1672-1676, i.e. the manuscripts presently being published in series VII of the Academy Edition , we have to deal with about 770 texts written on 1260 sheets of paper, of which nothing was published by Leibniz himself and only a small part prepared for eventual publication. What is said here concerning the texts in general holds also true also for the figures (drawings). Most of them were hastily sketched in pen and ink, only few are partly or completely constructed, many contain corrections or alterations.

Characteristic problems in the edition of the figures

The following case study will show some typical problems in editing Leibniz's mathematical drawings. Let us consider an unpublished manuscript from December 1674, *Problemata Methodi Tangentium inversae ad Geometricas constructiones reducta per applicationes curvarum*: in this manuscript Leibniz tries to apply his knowledge of the theory of evolutes and evolvents to problems in the inverse method of tangents.

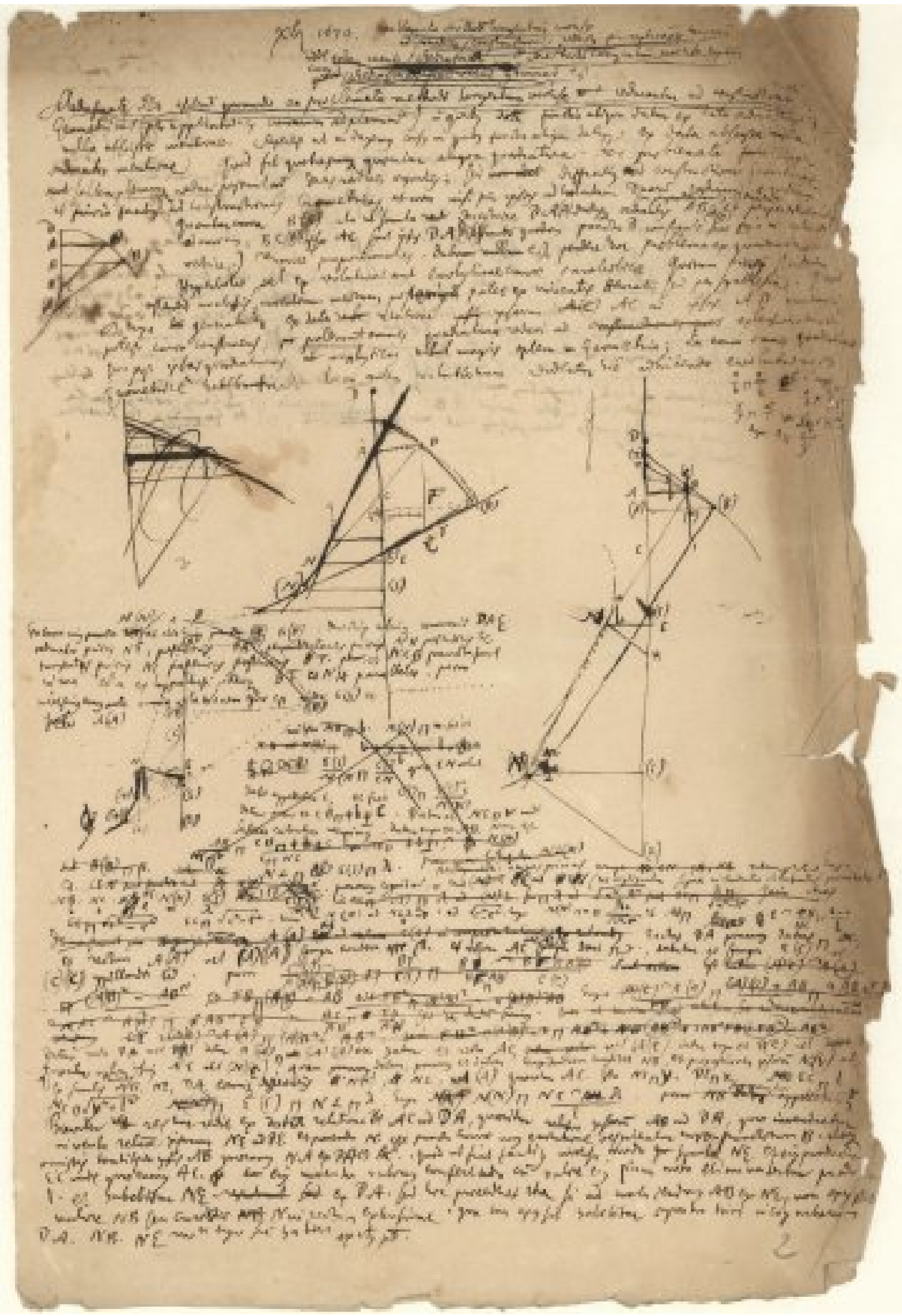


table1, fol. 2r

Obviously the middle part of the manuscript page contains a group of figures (drawings) of which the first is deleted. The two figures beside and below contain some roughly drawn lines, not drawn with ink but scratched into the paper. After the figures were drawn, some lines of text were written between them and on the figure below. The second part of this text is deleted. In the text written below the figures numerous deletions and corrections are visible.

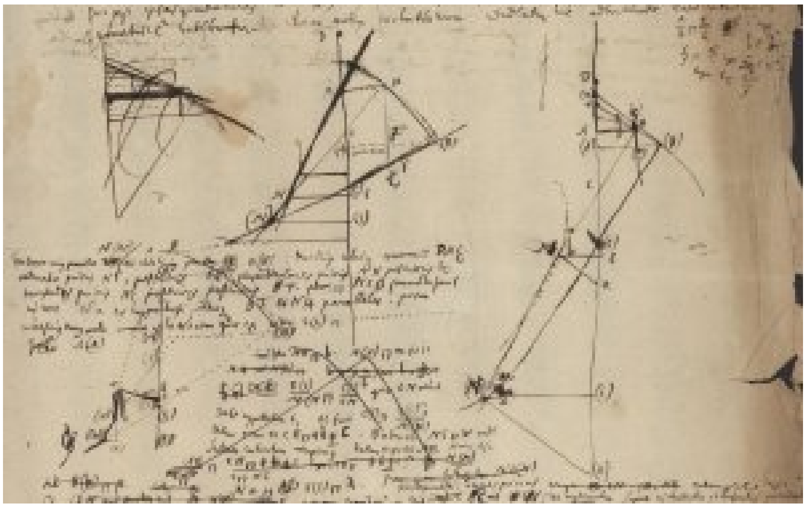
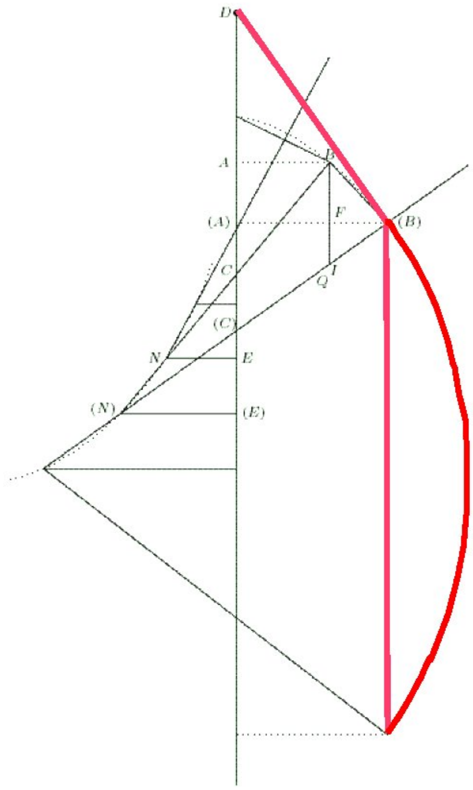


table 2, middle part of fol. 2r

Apparently the deleted figure is an aborted, incomplete drawing. This raises the first editorial problem to be solved: (1) Is it better to edit the figure or only to give a remark concerning the existence of such a deleted figure in the edition? A comparison of the three other figures shows that they represent the same situation, an evolute with its evolvent and the related tangents or normals respectively. There is some variation in the letters designating points, and in general the figure on the right side seems to be most elaborated. An analysis of the deletions in the text leads to the conclusion that the deleted text between the figures and a few others in the text below refer to the left figure, a longer deleted passage in the text below to the figure in the middle. However, Leibniz did not delete the two figures themselves. From these results a second editorial problem, (2) the treatment of these two figures. A possible solution for (2) seems to be, to put the two figures into the critical apparatus and to document this editorial intervention. Consequently, the several deletions related to the left figure have to be joined into one continuous textual variant. A third problem (3) in the edition of these figures is the treatment of the roughly drawn elements: Leibniz seldom used pencils for drawings, usually his figures are drawn with ink. Therefore he often scratched or pressed lines for auxiliary or preliminary constructions on the paper, a technique familiar in his or in former times. These roughly drawn lines are sometimes constructed with ruler and compass, sometimes freely drawn by hand. The editor has to decide whether -and eventually which- roughly drawn lines are used in the edition and how they are represented.



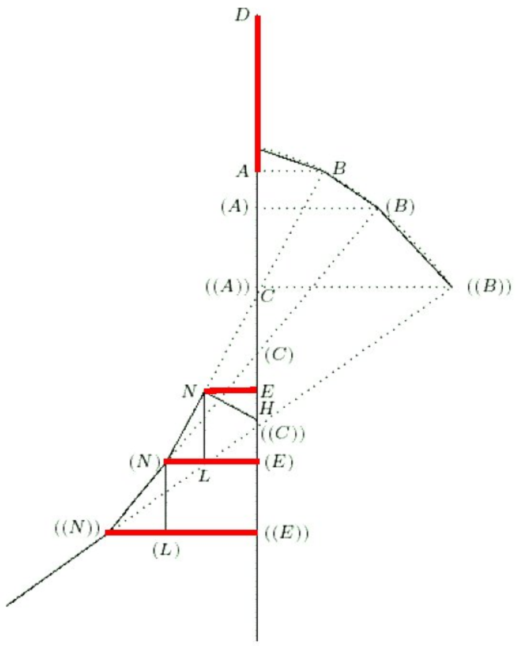


table 3 and 4, drawings with marked roughly drawn lines

The figure on the right side exemplifies another editorial problem (4), the alterations or corrections in figures, e.g. in the designation of points.

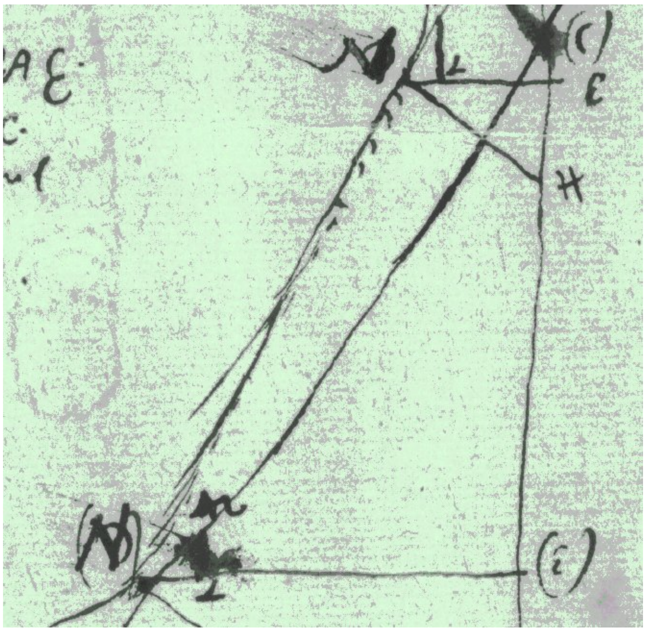


table 5, detail of right figure

Obviously, in this figure Leibniz first designated the points N and (N) by D and (D). This is somewhat strange, because in the other, probably earlier drawn figures, the corresponding points are already designated by N etc. and the designation D is used for the upper termination of the axis. Corresponding corrections can be found in the first line of the text referring to the right figure. The reason for this intermediate change of designation becomes clear by the content of a deleted text below that contains a reference to Huygens. Apparently Leibniz consulted the Horologium oscillatorium when working out the right figure which is modelled after the figure on p. 83 of Huygens' work. There the tangent point on the evolute is designated by D.

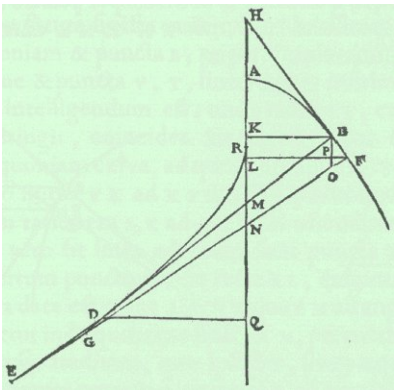


table 6, figure in Huygens

Of course, these few examples presented here are only a small selection from the variety of editorial problems posed by the drawings in Leibniz's mathematical papers. Leibniz often deleted, corrected or reworked figures or - as in our case - worked with varying sketches of figures. Sometimes there are elements (lines or points) in a figure that are not referred to within the accompanying text. Or there are texts that refer to drawings in other manuscripts that do not exist any more or simply cannot (yet) be identified. In addition, there some technical peculiarities that are quite common in Leibniz's lifetime: Since paper was expensive people tried to use it in an economical way, e.g. by roughly drawing lines in the way mentioned above. Another means of economy was to draw very small figures. From these result problems of interpretation of lines and points, especially when they are not determined by the text. Frequent corrections also pose similar difficulties.

Solutions in the academy edition

The conventions used in the academy edition suggest the following decisions or solutions for problems (1)-(4): (1) Whether a deleted figure will be edited depends largely on an evaluation of the mathematical content. In the case considered the deleted figure will only be mentioned in a footnote. (2) The solution mentioned above is not very economical for a print edition, but there is a more important reason to choose a different solution and to print the three figures consecutively (numbered Fig. 2a, 2b, and 2c) within the text: An examination of the text shows that no text related to these figures was written before the right figure was drawn and then corrected together with the first line of the text. The lines between the figures are not additions to the text below, but were written beforehand. The solution considered first would suggest a linearity not present in the text and so distort the provisional character of the text. (3) Roughly drawn lines are not presented in the edition, if they are replaced by lines drawn with ink etc. or if they are only auxiliary in a purely technical sense. Usually the roughly drawn lines are not distinguished in print from the other lines but are indicated only in the subscription of the figure. (4) In the case considered, the changing of the designation of points is annotated in a footnote because of the reference to the figure of Huygens. Without such a concrete reason, such changes are unlikely to be mentioned or to be mentioned only in a summary in the description of the manuscript (e.g. "many corrections in Fig. 2c").

Conclusion

Although I would argue that the choice of solutions adopted for series VII (Mathematical Papers) of the Academy Edition is a reasonable one, I think that it cannot provide a general model for establishing a critical method in the editing of mathematical diagrams. My main reason for thinking this is because in my view it is too restricted: partly by the economical and practical needs of a traditional print edition and partly by the general guidelines of an edition where mathematics is only a part of the whole project.

B.1.11 H. Hecht, *The Dynamics of Leibnitian Drawings and some Reflections on its Representation*

Preliminary remarks

In series VIII of the Academy-Edition Gottfried Wilhelm Leibniz: *Sämtliche Schriften und Briefe* the scientific, medical and technical manuscripts will be prepared for publication. These papers generally consist of two parts, of a written text on the one hand and of drawings and diagrams on the other hand. The drawings are not only illustrations of the written text. Indeed they play an important role for the understanding of the paper. In terms of quantum mechanics we can say that the two parts are complementarily to one another. In order to be able to represent the specific structure of our manuscripts by electronic means, we developed a program that allows a genetic presentation. It works very well with regard to the written text, but we are still searching for equivalent solutions regarding the drawings and diagrams. In the following I focus on three problems, which are appropriate to summarize our actual results as well as some problems to be solved.

Example 1

In general we represent the dynamics of the generation of drawings by the help of simple animations. The procedure does not make any problem if the drawing consists of only two or three generation layers or steps. In this case the final state of a drawing results from a cumulative process mostly beginning with a so-called blind drawing, i. e. a part of the drawing that Leibniz carved into the leaf using a circle or a needle, and ending with the transmitted ink drawing.

The situation changes if we are confronted with more than three of such steps. I will explain it referring to a technical manuscript that (in our reconstruction) contains a drawing consisting of five layers. Only two of them, namely, the first and the final state of the drawing are well defined. In figure 1 I have selected two steps of our animation. The left drawing describes the third layer and the right one the final state.

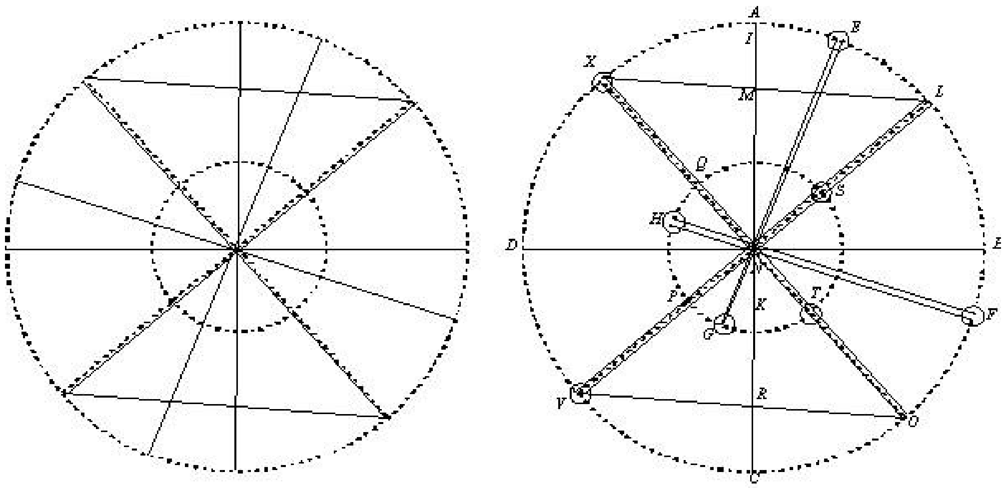


Fig.1

As it can be seen from the figure, Leibniz has deleted parts of the drawing, namely, the line segments from G to the periphery of the circle and from H to the periphery. Such a situation often occurs and it characterises a general problem of genetic reconstructions. Namely, we cannot decide exactly in any case if a certain deletion has to be assigned to the one state or to another. In our example Leibniz could have deleted the part in question at the fourth step or at the final steps. A deletion after the last one is also possible. Consequently, we have to accept a specific uncertainty if we try to give a genetic reconstruction of drawings. But how can we represent it? Our program *Leibniz online* does not make any problem to handle such situations concerning the written text, and it is our request for help to find a solution with regard to the drawings, too. I mean a solution that combines animations with the possibility to select one sequence of generation states from a set of possible sequences.

Example 2

My second example belongs to the convolute of optical manuscripts. In order to be sure about the optical path of a light ray, Leibniz designs a lot of drawings, all of them describing the refraction of a light ray running from an optical medium of higher density to an optical medium of lower density or vice versa. Often the drawings are only figurative versions of one and the same optical path problem, and we decided to regard them as steps of a cumulative process that finally delivers a valid drawing or something like that. For that reason we apply the same above-mentioned procedure of animations to represent the original. We are aware that the two situations we are describing with animations are quite different from one another. In our first example we dealt with the growth of only one drawing. Now we combine different drawings regarding them as steps of one and the same process. Our justification for such a procedure has something to do with the preliminary remarks. I have stressed the complementary relationship between written text and drawings. It means that Leibniz develops his ideas on two levels. He thinks, so to speak, writing down a text and he thinks in drawings or images. Therefore, in our opinion the two examples are to be understood as versions of one and the same creative process to find a solution for a scientific problem by means of drawings.

Example 3

It is an interesting result of our present work that there are drawings within the Leibnitian manuscripts which have been absolutely unknown up to now. In contrast to the just discussed drawings the dynamics of those ones cannot be described as a cumulative generation. In such a drawing Leibniz tries to represent a motion by a geometric structure. He tries to transform a temporal change into a spatial figure. My last example refers to Leibniz' investigations of the air pressure. In one of his manuscripts (see the picture at the left hand site) he discusses an above closed tube. The tube contains air and mercury in such a way that a grafting of mercury presses the air column downwards. It is clear that the position of the grafting of mercury depends on the inclination of the tube with regard to the horizontal plane. Supposed, the angle position between the tube and the plane is 90 degrees, then, the mercury grafting will be at its lowest position. As it can be seen from Leibniz' drawing, he marks up different position of the grafting, among them 0 degree and 90 degrees. But 90 degrees in his drawing do not correspond to the perpendicular position of the tube and 0 degree not to the horizontal position. In our interpretation Leibniz does not want to identify a well defined positions by his notation, but a sequence of possible inclinations of the tube. At least it is a motion that he has in mind, beginning with the perpendicular and ending at the horizontal position. We, therefore, represent it by an animation that goes step by step from inclination to inclination.

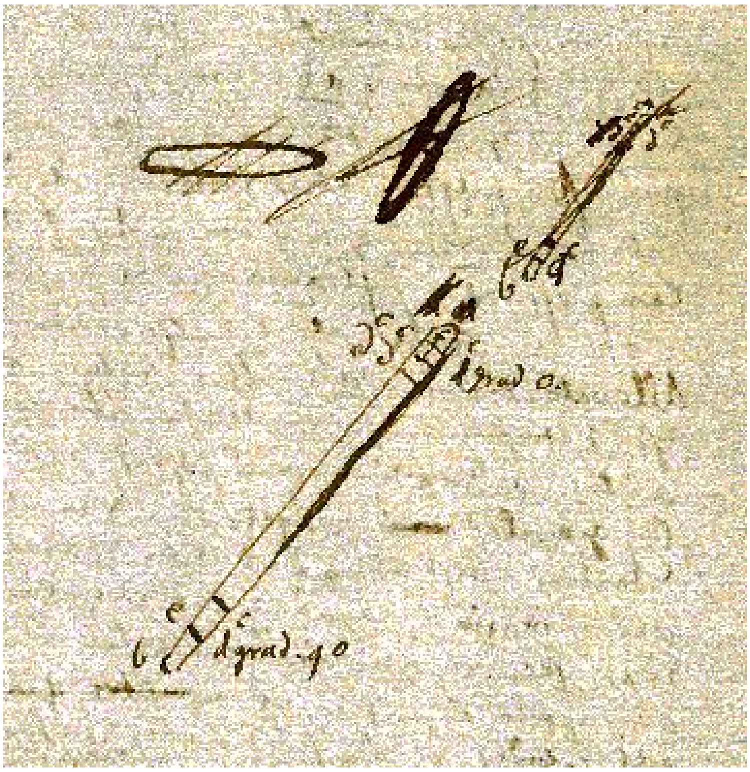


Fig.2

A final remark It seems to me that animations as we use them are helpful means to reconstruct the different dynamic implications of Leibnitian drawings. Nevertheless, they have to be ameliorated to represent the discussed examples more adequately as we actually can do it.

B.1.12 O. Besomi, *Riflessioni intorno all'edizione delle figure che accompagnano il "Saggiatore" di Galileo Galilei*

I testi. Premessa

Tra gli eventi astronomici straordinari della prima metà del Seicento, il più spettacolare fu senz'altro la comparsa, alla fine di novembre del 1618, di una "grandissima" cometa, osservata nel suo spostamento, nel corso dei due mesi successivi, dalla costellazione della Bilancia fino a quella dell'Orsa Maggiore. Questa cometa, oggi dagli astronomi denominata "1618 II", preceduta da due altri fenomeni analoghi, ma molto meno appariscenti, suscitò non solo meraviglia, ma angosciati timori e interrogativi nella popolazione e nelle corti. Nel contesto del dibattito scientifico e dottrinale cruciale di allora, vertente sul sistema copernicano, la grande cometa offriva agli astronomi (fossero essi anticopernicani o copernicani) un motivo di massima attenzione: la speranza di poter individuare, in quel peculiare fenomeno, degli indizi che potessero essere assunti a sostegno della tesi dell'immobilità della terra o della sua mobilità. Non meno rilevante era, d'altro verso, il dibattito quanto alla collocazione e alla natura delle comete.

La polemica intorno alle comete è ancorata, per quanto riguarda Galileo, alle opere seguenti:

1. ORAZIO GRASSI, Disputatio astronomica De tribus cometis, Roma, febbraio-marzo del 1619;

2. GALILEO-GUIDUCCI, Discorso sulle comete, Firenze, giugno 1619
3. ORAZIO GRASSI, Libra, Perugia 1619
4. GALILEO, Postille di Galileo alla Libra
5. GALILEO, Il Saggiatore, Roma 1623

Il *Saggiatore* (Roma 1623) nasce come replica dello scienziato pisano alla Libra astronomica ac philosophica (Perugia 1619) di Lotario Sarsi (pseudonimo del gesuita Orazio Grassi). Dedicato a Urbano VIII appena assunto al soglio pontificio, il *Saggiatore* è il libro con il quale Galileo riappare in prima persona sulla scena culturale, dopo il monito del 1616 che vietava allo scienziato di professare o divulgare la teoria copernicana. La Libra del Grassi è riprodotta nel *Saggiatore* integralmente quanto a testo e a disegni, in modo articolato, secondo porzioni semanticamente compatte che Galileo sottopone a esame, sempre rispettandone rigorosamente l'ordine sintagmatico.

I Disegni

Delle 18 figure che accompagnano il *Saggiatore*, ben 15, a illustrazione del testo latino del Sarsi (Grassi), erano già inserite nel volume della Libra; Galileo le riproduce all'interno del testo che trascrive nel suo libro per farne oggetto di osservazioni critiche. Solo tre disegni (ai ğğ 11, 16*; 24, 31; 49, 87*) sono originali dell'autore del *Saggiatore*. Galileo rifà fedelmente le illustrazione del Sarsi, in scala 1:1, con minimi scarti, e li colloca al luogo opportuno, tenendo conto, ovviamente, dei condizionamenti dati da uno specchio di pagina che non coincide con quello della Libra. La tabella che segue rende conto dell'esatta situazione:

	Saggiatore	Princeps	Libra	GALILEO Opere
1	6, 21	22	-	VI 230
2	11, 16*	41	-	VI 243
3	24, 3*	114	28	VI 141
4	24, 6*	114	28	VI 141
5	24, 8*	114	29	VI 141
6	24, 31	118	-	VI 298
7	25, 7*	120	31	VI 143
8	30, 6*	130	35	VI 146
9	31, 11*	132	37	VI 147
10	39, 3*	157	46	VI 155
11	40, 9*	161	48	VI 156
12	40, 22*	163	50	VI 158
13	40, 24*	163	50	VI 158
14	40, 27*	164	51	VI 159
15	40, 32*	164	52	VI 159
16	49, 23*	204	63	VI 171
17	49,26*	205	63	VI 171
18	49, 87*	217	-	VI 361

Varie soluzione ecdotiche dei disegni del *Saggiatore*

- L'Albèri (GALILEO, Le opere, Firenze 1842) ha riproposto il testo (e disegni) del Sarsi integralmente nelle due sedi che loro convengono, nella Libra e all'interno del *Saggiatore*.
- Nell'Edizione Nazionale (Firenze, 1890-1909), Favaro ha scorporato il testo della Libra dal *Saggiatore*, scegliendo, per ragioni pratiche, una soluzione sulla quale egli stesso ha nutrito dubbi. Ne risulta un'edizione del *Saggiatore* ampiamente amputata, disorganica. La situazione è la seguente: a) viene restituita la porzione di testo in volgare, ossia il testo di Galileo b) il testo latino della Libra (dato integralmente, per spezzoni, da Galileo), viene estrapolato dal *Saggiatore*, dato in altra sede del volume e ricomposto come unità Libra. Di conseguenza: c) i disegni originali di Grassi, riprodotti con il testo nel *Saggiatore*, accompagnano il testo di Grassi, e non figurano in quello di Galileo d) gli unici disegni che vengono dati nella mutilata edizione di Favaro sono i tre originali di Galileo.
- L'edizione Besomi-Helbing (Antenore, Roma-Padova 2004)

  La situazione illustrata permette di riflettere - sulla soluzione da dare alle figure in una edizione critica di un testo che le comprende come parte integrante - su aspetti ecdotici delle figure (siano esse di natura geometrica, o chiamate ad illustrare fenomeni di natura astronomica o altra), avendo a disposizione - un autografo (con o senza apografi), in una o più redazioni - testimoni apografi manoscritti e/o a stampa - sul loro trattamento e sulla loro posizione nel contesto del trattato.